Package ‘qrng’

February 29, 2024

Version 0.0-10

Encoding UTF-8

Title (Randomized) Quasi-Random Number Generators

Description Functionality for generating (randomized) quasi-random numbers in high dimensions.

Author Marius Hofert [aut, cre],
Christiane Lemieux [aut]

Maintainer Marius Hofert <mhofert@hku.hk>

Depends R (>= 3.0.0)

Imports utils

Suggests spacefillr, randtoolbox, copula

Enhances

License GPL-2 | GPL-3

NeedsCompilation yes

Repository CRAN

Date/Publication 2024-02-29 14:40:02 UTC

R topics documented:

qrng *Compute Quasi-Random Sequences*

Description

Computing Korobov, generalize Halton and Sobol' quasi-random sequences.

Usage

```
korobov(n, d = 1, generator, randomize = c("none", "shift"))
ghalton(n, d = 1, method = c("generalized", "halton"))
sobol  (n, d = 1, randomize = c("none", "digital.shift", "Owen", "Faure.Tezuka",
                                "Owen.Faure.Tezuka"), seed, skip = 0, ...)
```

Arguments

`n` number n of points to be generated ≥ 2.

`d` dimension d.

`generator` `numeric` of length d or length 1 (in which case it is appropriately extended to length d). All numbers must be in $\{1, \dots, n\}$ and must be (coercible to) integers.

`randomize` `character` string indicating the type of randomization for the point set.

- `korobov()` one of the following:
  - **"none"** no randomization.
  - **"shift"** a uniform random variate modulo 1.
- `sobol()` one of the following:
  - `"none"` no randomization.
  - **"digital.shift"** a digital shift.
  - **"Owen"** calls `generate_sobol_owen_set()` from package **spacefillr**.
  - **"Faure.Tezuka","Owen.Faure.Tezuka"** calls `sobol()` from package **randtoolbox** with `scrambling` being 2 and 3, respectively.

  If `randomize` is a `logical`, then it is interpreted as `"none"` if `FALSE` and `"digital.shift"` if `TRUE`.

`method` `character` string indicating which sequence is generated, generalized Halton or (plain) Halton.

`seed` if provided, an integer used within `set.seed()` for the non-scrambling `randomize` methods (so `"none"` or `"digital.shift"`) or passed to the underlying `generate_sobol_owen_set()` from package **spacefillr** (for `"Owen"`) and `sobol()` from package **randtoolbox** for the scrambling methods. If not provided, the non-scrambling methods respect a global `set.seed()` but the scrambling methods do not (they then use a randomly generated one based on time and process identifier).

`skip` number of initial points in the sequence to be skipped (`skip = 0` means the sequence starts with at the origin). Note that for the scrambling methods this simply computes `n + skip` points and omits the first `skip`-many.

`...` additional arguments passed to `sobol()` from package **randtoolbox** for randomization methods `"Faure.Tezuka"` and `"Owen.Faure.Tezuka"`.

Details

For `sobol()` examples see `demo(sobol_examples)`.

Note that these procedures call fast C code. The following restrictions apply:

korobov() `n,d` must be $\leq 2^{31} - 1$.

ghalton() `n` must be $\leq 2^{32} - 1$ and `d` must be ≤ 360.

sobol() if `randomize = "none"` or `randomize = "digital.shift"`, `n` must be $\leq 2^{31} - 1$ and `d` must be ≤ 16510.

The choice of parameters for `korobov()` is crucial for the quality of this quasi-random sequence (only basic sanity checks are conducted). For more details, see l'Ecuyer and Lemieux (2000).

The generalized Halton sequence uses the scrambling factors of Faure and Lemieux (2009).

Value

`korobov()` and `ghalton()` return an (n, d)-`matrix`; for $d = 1$ an n-vector is returned.

Author(s)

Marius Hofert and Christiane Lemieux

References

Faure, H., Lemieux, C. (2009). Generalized Halton Sequences in 2008: A Comparative Study. *ACM-TOMACS* **19**(4), Article 15.

l'Ecuyer, P., Lemieux, C. (2000). Variance Reduction via Lattice Rules. *Stochastic Models and Simulation*, 1214–1235.

Lemieux, C., Cieslak, M., Luttmer, K. (2004). RandQMC User's guide. See https://www.math.uwaterloo.ca/~clemieux/randq

Examples

```
n <- 1021 # prime
d <- 4 # dimension

## Korobov's sequence
generator <- 76 # see l'Ecuyer and Lemieux
u <- korobov(n, d = d, generator = generator)
pairs(u, gap = 0, pch = ".", labels = as.expression(
      sapply(1:d, function(j) bquote(italic(u[.(j)])))))

## Randomized Korobov's sequence
set.seed(271)
u <- korobov(n, d = d, generator = generator, randomize = "shift")
pairs(u, gap = 0, pch = ".", labels = as.expression(
      sapply(1:d, function(j) bquote(italic(u[.(j)])))))

## Generalized Halton sequence (randomized by definition)
set.seed(271)
u <- ghalton(n, d)
```

```
pairs(u, gap = 0, pch = ".", labels = as.expression(
      sapply(1:d, function(j) bquote(italic(u[.(j)])))))

## For sobol() examples, see demo(sobol_examples)
```

`test_functions` *Test Functions*

Description

Functions for testing low-discrepancy sequences.

Usage

```
sum_of_squares(u)
sobol_g(u, copula = copula::indepCopula(dim = ncol(u)), alpha = 1:ncol(u), ...)
exceedance(x, q, p = 0.99, method = c("indicator", "individual.given.sum.exceeds",
                                      "sum.given.sum.exceeds"))
```

Arguments

u	(n, d)-matrix containing n d-dimensional realizations (of a potential quasi-random number generator). For `sum_of_squares()` these need to be marginally standard uniform and for `sobol_g()` they need to follow the copula specified by `copula`.
copula	`Copula` object for which the inverse Rosenblatt transformation exists.
alpha	vector of parameters of Sobol's g test function.
...	additional arguments passed to the underlying `cCopula()`.
x	(n, d)-matrix containing n d-dimensional realizations.
q	**"indicator"** d-vector containing the componentwise thresholds; if a number it is recycled to a d-vector. **"individual.given.sum.exceeds", "sum.given.sum.exceeds"** threshold for the sum (row sums of `x`).
p	If `q` is not provided, the probability `p` is used to determine `q`. **"indicator"** d-vector containing the probabilities determining componentwise thresholds via empirical quantiles; if a number, it is recycled to a d-vector. **"individual.given.sum.exceeds", "sum.given.sum.exceeds"** probability determining the threshold for the sum (row sums of `x`) via the corresponding empirical quantile.
method	`character` string indicating the type of exceedance computed (see Section Value below).

Details

For examples see the demo `man_test_functions`.

See `ES_np(<matrix>)` from **qrmtools** for another test function.

Value

sum_of_squares() returns an n-vector (numeric(n)) with the rowwise computed scaled sum of squares (theoretically integrating to 1).

sobol_g() returns an n-vector (numeric(n)) with the rowwise computed Sobol' g functions.

exceedance()'s return value depends on method:

"indicator" returns indicators whether, componentwise, x exceeds the threshold determined by q.

"individual.given.sum.exceeds" returns all rows of x whose sum exceeds the threshold determined by q.

"sum.given.sum.exceeds" returns the row sums of those rows of x whose sum exceeds the threshold determined by q.

Author(s)

Marius Hofert and Christiane Lemieux

References

Radovic, I., Sobol', I. M. and Tichy, R. F. (1996). Quasi-Monte Carlo methods for numerical integration: Comparison of different low discrepancy sequences. *Monte Carlo Methods and Applications* **2**(1), 1–14.

Faure, H., Lemieux, C. (2009). Generalized Halton Sequences in 2008: A Comparative Study. *ACM-TOMACS* **19**(4), Article 15.

Owen, A. B. (2003). The dimension distribution and quadrature test functions. *Stat. Sinica* **13**, 1-–17.

Sobol', I. M. and Asotsky, D. I. (2003). One more experiment on estimating high-dimensional integrals by quasi-Monte Carlo methods. *Math. Comput. Simul.* **62**, 255—263.

Examples

```
## Generate some (here: copula, pseudo-random) data
library(copula)
set.seed(271)
cop <- claytonCopula(iTau(claytonCopula(), tau = 0.5)) # Clayton copula
U <- rCopula(1000, copula = cop)

## Compute sum of squares test function
mean(sum_of_squares(U)) # estimate of E(3(sum_{j=1}^d U_j^2)/d)

## Compute the Sobol' g test function
if(packageVersion("copula") >= "0.999-20")
    mean(sobol_g(U)) # estimate of E(<Sobol's g function>)

## Compute an exceedance probability
X <- qnorm(U)
mean(exceedance(X, q = qnorm(0.99))) # fixed threshold q
mean(exceedance(X, p = 0.99)) # empirically estimated marginal p-quantiles as thresholds
```

```
## Compute 99% expected shortfall for the sum
mean(exceedance(X, p = 0.99, method = "sum.given.sum.exceeds"))
## Or use ES_np(X, level = 0.99) from 'qrmtools'
```

---

to_array *Compute Matrices to Arrays*

Description

Converting higher-dimensional matrices of quasi-random numbers to arrays of specific formats.

Usage

```
to_array(x, f, format = c("(n*f,d)", "(n,f,d)"))
```

Arguments

x	(n, fd)-matrix of quasi-random numbers to be converted.
f	factor $f \geq 1$ dividing `ncol{x}`.
format	`character` string indicating the output format to which `x` should be converted.

Details

`to_array()` is helpful for converting quasi-random numbers to time series paths.

Value

`(n * f, d)`-`matrix` or `(n, f, d)`-`array` depending on the chosen `format`.

Author(s)

Marius Hofert

See Also

`korobov()`, `ghalton()`, `sobol()`.

Examples

```
## Basic call
N <- 4 # replications
n <- 3 # time steps
d <- 2 # dimension
set.seed(271) # note: respected for the choice of 'randomize'
x <- sobol(N, d = n * d, randomize = "digital.shift") # higher-dim. Sobol'
stopifnot(dim(to_array(x, f = n)) == c(N * n, d)) # conversion and check
stopifnot(dim(to_array(x, f = n, format = "(n,f,d)")) == c(N, n, d))

## See how the conversion is done
```

```
(x <- matrix(1:(N * n * d), nrow = N, byrow = TRUE))
to_array(x, f = n) # => (n * d)-column x was blocked in n groups of size d each
```

Index