Package ‘qfa’

December 20, 2024

Type Package

Title Quantile-Frequency Analysis (QFA) of Time Series

Version 3.1

Date 2024-12-16

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Description
Quantile-frequency analysis (QFA) of time series based on trigonometric quantile regression.
References:
[1] Li, T.-H. (2012) ``Quantile periodograms'', Journal of the American Statistical Association, 107, 765–776, <doi:10.1080/01621459.2012.682815>.
[2] Li, T.-H. (2014) Time Series with Mixed Spectra, CRC Press, <doi:10.1201/b15154>
[3] Li, T.-H. (2022) ``Quantile Fourier transform, quantile series, and nonparametric estimation of quantile spectra'', <doi:10.48550/arXiv.2211.05844>.
[4] Li, T.-H. (2024) ``Quantile crossing spectrum and spline autoregression estimation,'' <doi:10.48550/arXiv.2412.02513>.

Depends R (>= 3.5)

Imports RhpcBLASctl, doParallel, fields, foreach, mgcv, nlme, parallel, quantreg, splines, stats, graphics, colorRamps, MASS

License GPL (>= 2)

URL https://github.com/IBM/qfa, https://github.com/thl2019/QFA

NeedsCompilation yes

Encoding UTF-8

RoxygenNote 7.3.2

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Repository CRAN

Date/Publication 2024-12-20 09:40:01 UTC

Contents

`per` *Periodogram (PER)*

Description

This function computes the periodogram or periodogram matrix for univariate or multivariate time series.

Usage

```
per(y)
```

Arguments

y	vector (n) or matrix (n x nc) of time series

Value

A vector (n) or array (nc x nc x n) of periodogram

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.per <- per(y)
plot(y.per)
```

---

qacf *Quantile Autocovariance Function (QACF)*

Description

This function computes quantile autocovariance function (QACF) from time series or quantile discrete Fourier transform (QDFT).

Usage

```
qacf(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`y.qdft`	matrix or array of pre-calculated QDFT (default = `NULL`: compute from `y` and `tau`); if `y.qdft` is supplied, `y` and `tau` can be left unspecified
`n.cores`	number of cores for parallel computing of QDFT if `y.qdft` = `NULL` (default = 1)
`cl`	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

matrix or array of quantile autocovariance function

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qacf <- qacf(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qacf <- qacf(y.qdft=y.qdft)
```

qcser — *Quantile-Crossing Series (QCSER)*

Description

This function creates the quantile-crossing series (QCSER) for univariate or multivariate time series.

Usage

```
qcser(y, tau, normalize = FALSE)
```

Arguments

y	vector or matrix of time series
tau	vector of quantile levels in (0,1)
normalize	TRUE or FALSE (default): normalize QCSER to have unit variance

Value

A matrix or array of quantile-crossing series

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qser <- qcser(y,tau)
dim(y.qser)
```

qdft — *Quantile Discrete Fourier Transform (QDFT)*

Description

This function computes quantile discrete Fourier transform (QDFT) for univariate or multivariate time series.

Usage

```
qdft(y, tau, n.cores = 1, cl = NULL)
```

Arguments

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix or array of quantile discrete Fourier transform of y

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y,tau)
# Make a cluster for repeated use
n.cores <- 2
cl <- parallel::makeCluster(n.cores)
parallel::clusterExport(cl, c("tqr.fit"))
doParallel::registerDoParallel(cl)
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y1,tau,n.cores=n.cores,cl=cl)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y2,tau,n.cores=n.cores,cl=cl)
parallel::stopCluster(cl)
```

---

qdft2qacf *Quantile Autocovariance Function (QACF)*

Description

This function computes quantile autocovariance function (QACF) from QDFT.

Usage

```
qdft2qacf(y.qdft, return.qser = FALSE)
```

Arguments

y.qdft	matrix or array of QDFT from qdft()
return.qser	if TRUE, return quantile series (QSER) along with QACF

Value

matrix or array of quantile autocovariance function if return.sqer = FALSE (default), else a list with the following elements:

qacf	matirx or array of quantile autocovariance function
qser	matrix or array of quantile series

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qacf <- qdft2qacf(y.qdft)
plot(c(0:9),y.qacf[c(1:10),1],type='h',xlab="LAG",ylab="QACF")
y.qser <- qdft2qacf(y.qdft,return.qser=TRUE)$qser
plot(y.qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qacf <- qdft2qacf(y.qdft)
plot(c(0:9),y.qacf[1,2,c(1:10),1],type='h',xlab="LAG",ylab="QACF")
```

---

qdft2qper — *Quantile Periodogram (QPER)*

Description

This function computes quantile periodogram (QPER) from QDFT.

Usage

```
qdft2qper(y.qdft)
```

Arguments

`y.qdft`	matrix or array of QDFT from `qdft()`

Value

matrix or array of quantile periodogram

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qper <- qdft2qper(y.qdft)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(y.qper[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qper <- qdft2qper(y.qdft)
```

```
qfa.plot(ff[sel.f],tau,Re(y.qper[1,1,sel.f,]))
qfa.plot(ff[sel.f],tau,Re(y.qper[1,2,sel.f,]))
```

---

qdft2qser *Quantile Series (QSER)*

Description

This function computes quantile series (QSER) from QDFT.

Usage

```
qdft2qser(y.qdft)
```

Arguments

y.qdft	matrix or array of QDFT from qdft()

Value

matrix or array of quantile series

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qser <- qdft2qser(y.qdft)
plot(y.qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qser <- qdft2qser(y.qdft)
plot(y.qser[1,,1],type='l',xlab="TIME",ylab="QSER")
```

---

qfa.plot *Quantile-Frequency Plot*

Description

This function creates an image plot of quantile spectrum.

Usage

```
qfa.plot(
  freq,
  tau,
  rqper,
  rg.qper = range(rqper),
  rg.tau = range(tau),
  rg.freq = c(0, 0.5),
  color = colorRamps::matlab.like2(1024),
  ylab = "QUANTILE LEVEL",
  xlab = "FREQUENCY",
  tlab = NULL,
  set.par = TRUE,
  legend.plot = TRUE
)
```

Arguments

`freq`	sequence of frequencies in (0,0.5) at which quantile spectrum is evaluated
`tau`	sequence of quantile levels in (0,1) at which quantile spectrum is evaluated
`rqper`	real-valued matrix of quantile spectrum evaluated on the `freq` x `tau` grid
`rg.qper`	`zlim` for `qper` (default = `range(qper)`)
`rg.tau`	`ylim` for `tau` (default = `range(tau)`)
`rg.freq`	`xlim` for `freq` (default = `c(0,0.5)`)
`color`	colors (default = `colorRamps::matlab.like2(1024)`)
`ylab`	label of y-axis (default = `"QUANTILE LEVEL"`)
`xlab`	label of x-axis (default = `"FREQUENCY"`)
`tlab`	title of plot (default = `NULL`)
`set.par`	if `TRUE`, `par()` is set internally (single image)
`legend.plot`	if `TRUE`, legend plot is added

Value

no return value

`qkl.divergence` *Kullback-Leibler Divergence of Quantile Spectral Estimate*

Description

This function computes Kullback-Leibler divergence (KLD) of quantile spectral estimate.

Usage

```
qkl.divergence(y.qper, qspec, sel.f = NULL, sel.tau = NULL)
```

Arguments

`y.qper`	matrix or array of quantile spectral estimate from, e.g., `qspec.lw()`
`qspec`	matrix of array of true quantile spectrum (same dimension as `y.qper`)
`sel.f`	index of selected frequencies for computation (default = `NULL`: all frequencies)
`sel.tau`	index of selected quantile levels for computation (default = `NULL`: all quantile levels)

Value

real number of Kullback-Leibler divergence

`qper` *Quantile Periodogram (QPER)*

Description

This function computes quantile periodogram (QPER) from time series or quantile discrete Fourier transform (QDFT).

Usage

```
qper(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`y.qdft`	matrix or array of pre-calculated QDFT (default = `NULL`: compute from `y` and `tau`); if `y.qdft` is supplied, `y` and `tau` can be left unspecified
`n.cores`	number of cores for parallel computing of QDFT if `y.qdft` = `NULL` (default = 1)
`cl`	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

matrix or array of quantile periodogram

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qper <- qper(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qper <- qper(y.qdft=y.qdft)
```

---

qper2 *Quantile Periodogram Type II (QPER2)*

Description

This function computes type-II quantile periodogram for univariate time series.

Usage

```
qper2(y, freq, tau, weights = NULL, n.cores = 1, cl = NULL)
```

Arguments

`y`	univariate time series
`freq`	sequence of frequencies in [0,1)
`tau`	sequence of quantile levels in (0,1)
`weights`	sequence of weights in quantile regression (default = `NULL`: weights equal to 1)
`n.cores`	number of cores for parallel computing (default = 1)
`cl`	pre-existing cluster for repeated parallel computing (default = `NULL`)

Value

matrix of quantile periodogram evaluated on `freq * tau` grid

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper2 <- qper2(y,ff,tau)
qfa.plot(ff[sel.f],tau,Re(y.qper2[sel.f,]))
```

qser — *Quantile Series (QSER)*

Description

This function computes quantile series (QSER) from time series or quantile discrete Fourier transform (QDFT).

Usage

```
qser(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`y.qdft`	matrix or array of pre-calculated QDFT (default = `NULL`: compute from `y` and `tau`); if `y.qdft` is supplied, `y` and `tau` can be left unspecified
`n.cores`	number of cores for parallel computing of QDFT if `y.qdft` = `NULL` (default = 1)
`cl`	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

matrix or array of quantile series

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qser <- qser(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qser <- qser(y.qdft=y.qdft)
```

qser2ar — *Autoregression (AR) Model of Quantile Series*

Description

This function fits an autoregression (AR) model to quantile series (QSER) separately for each quantile level using `stats::ar()`.

Usage

```
qser2ar(y.qser, p = NULL, order.max = NULL, method = c("none", "gamm", "sp"))
```

Arguments

`y.qser`	matrix or array of pre-calculated QSER, e.g., using `qser()`
`p`	order of AR model (default = `NULL`: selected by AIC)
`order.max`	maximum order for AIC if `p = NULL` (default = `NULL`: determined by `stats::ar()`)
`method`	quantile smoothing method: `"gamm"`, `"sp"`, or `"NA"` (default)

Value

a list with the following elements:

`A`	matrix or array of AR coefficients
`V`	vector or matrix of residual covariance
`p`	order of AR model
`n`	length of time series
`residuals`	matrix or array of residuals

---

qser2qacf — *ACF of Quantile Series (QSER) or Quantile-Crossing Series (QCACF)*

Description

This function creates the ACF of quantile series or quantile-crossing series

Usage

```
qser2qacf(y.qser)
```

Arguments

`y.qser`	matrix or array of quantile-crossing series

Value

A matrix or array of ACF

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qser <- qcser(y,tau)
y.qacf <- qser2qacf(y.qser)
dim(y.qacf)
```

qser2sar *Spline Autoregression (SAR) Model of Quantile Series*

Description

This function fits spline autoregression (SAR) model to quantile series (QSER).

Usage

```
qser2sar(
  y.qser,
  tau,
  d = 1,
  p = NULL,
  order.max = NULL,
  spar = NULL,
  method = c("GCV", "AIC", "BIC"),
  weighted = FALSE
)
```

Arguments

`y.qser`	matrix or array of pre-calculated QSER, e.g., using `qser()`
`tau`	sequence of quantile levels where `y.qser` is calculated
`d`	subsampling rate of quantile levels (default = 1)
`p`	order of SAR model (default = `NULL`: automatically selected by AIC)
`order.max`	maximum order for AIC if `p = NULL` (default = `NULL`: determined by `stats::ar()`)
`spar`	penalty parameter alla `smooth.spline` (default = `NULL`: automatically selected)
`method`	criterion for penalty parameter selection: `"AIC"` (default), `"BIC"`, or `"GCV"`
`weighted`	if `TRUE`, penalty function is weighted (default = `FALSE`)

Value

a list with the following elements:

`A`	matrix or array of SAR coefficients
`V`	vector or matrix of SAR residual covariance
`p`	order of SAR model
`spar`	penalty parameter
`tau`	sequence of quantile levels
`n`	length of time series
`d`	subsampling rate of quantile levels
`weighted`	option for weighted penalty function
`fit`	object containing details of SAR fit

qspec.ar *Autoregression (AR) Estimator of Quantile Spectrum*

Description

This function computes autoregression (AR) estimate of quantile spectrum from time series or quantile series (QSER).

Usage

```
qspec.ar(
  y,
  tau,
  y.qser = NULL,
  p = NULL,
  order.max = NULL,
  freq = NULL,
  method = c("none", "gamm", "sp"),
  n.cores = 1,
  cl = NULL
)
```

Arguments

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
y.qser	matrix or array of pre-calculated QSER (default = NULL: compute from y and tau);
p	order of AR model (default = NULL: automatically selected by AIC)
order.max	maximum order for AIC if p = NULL (default = NULL: determined by stats::ar())
freq	sequence of frequencies in [0,1) (default = NULL: all Fourier frequencies)
method	quantile smoothing method: "gamm" for mgcv::gamm(), "sp" for stats::smooth.spline(), or "none" (default) if y.qser is supplied, y and tau can be left unspecified
n.cores	number of cores for parallel computing of QDFT if y.qser = NULL (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = NULL)

Value

a list with the following elements:

spec	matrix or array of AR quantile spectrum
freq	sequence of frequencies
fit	object of AR model
qser	matrix or array of quantile series if y.qser = NULL

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y <- cbind(y1,y2)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qspec.ar <- qspec.ar(y,tau,p=1)$spec
qfa.plot(ff[sel.f],tau,Re(y.qspec.ar[1,1,sel.f,]))
y.qser <- qcser(y1,tau)
y.qspec.ar <- qspec.ar(y.qser=y.qser,p=1)$spec
qfa.plot(ff[sel.f],tau,Re(y.qspec.ar[sel.f,]))
y.qspec.arqs <- qspec.ar(y.qser=y.qser,p=1,method="sp")$spec
qfa.plot(ff[sel.f],tau,Re(y.qspec.arqs[sel.f,]))
```

---

qspec.lw — *Lag-Window (LW) Estimator of Quantile Spectrum*

Description

This function computes lag-window (LW) estimate of quantile spectrum with or without quantile smoothing from time series or quantile autocovariance function (QACF).

Usage

```
qspec.lw(
  y,
  tau,
  y.qacf = NULL,
  M = NULL,
  method = c("none", "gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`y.qacf`	matrix or array of pre-calculated QACF (default = `NULL`: compute from `y` and `tau`); if `y.qacf` is supplied, `y` and `tau` can be left unspecified
`M`	bandwidth parameter of lag window (default = `NULL`: quantile periodogram)
`method`	quantile smoothing method: `"gamm"` for `mgcv::gamm()`, `"sp"` for `stats::smooth.spline()`, or `"none"` (default)

spar	smoothing parameter in smooth.spline() if method = "sp" (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

Value

A list with the following elements:

spec	matrix or array of spectral estimate
spec.lw	matrix or array of spectral estimate without quantile smoothing
lw	lag-window sequence
qacf	matrix or array of quantile autocovariance function if y.qacf = NULL

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qacf <- qacf(cbind(y1,y2),tau)
y.qper.lw <- qspec.lw(y.qacf=y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.lw[1,1,sel.f,]))
y.qper.lwqs <- qspec.lw(y.qacf=y.qacf,M=5,method="sp",spar=0.9)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.lwqs[1,1,sel.f,]))
```

---

qspec.sar *Spline Autoregression (SAR) Estimator of Quantile Spectrum*

Description

This function computes spline autoregression (SAR) estimate of quantile spectrum.

Usage

```
qspec.sar(
  y,
  y.qser = NULL,
  tau,
  d = 1,
  p = NULL,
  order.max = NULL,
  spar = NULL,
  method = c("GCV", "AIC", "BIC"),
  weighted = FALSE,
  freq = NULL,
```

```
  n.cores = 1,
  cl = NULL
)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`y.qser`	matrix or array of pre-calculated QSER (default = `NULL`: compute from `y` and `tau`); if `y.qser` is supplied, `y` can be left unspecified
`tau`	sequence of quantile levels in (0,1)
`d`	subsampling rate of quantile levels (default = 1)
`p`	order of SAR model (default = `NULL`: automatically selected by AIC)
`order.max`	maximum order for AIC if `p` = `NULL` (default = `NULL`: determined by `stats::ar()`)
`spar`	penalty parameter alla `smooth.spline` (default = `NULL`: automatically selected)
`method`	criterion for penalty parameter selection: `"GCV"`, `"AIC"` (default), or `"BIC"`
`weighted`	if `TRUE`, penalty function is weighted (default = `FALSE`)
`freq`	sequence of frequencies in [0,1) (default = `NULL`: all Fourier frequencies)
`n.cores`	number of cores for parallel computing of QDFT if `y.qser` = `NULL` (default = 1)
`cl`	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

a list with the following elements:

`spec`	matrix or array of SAR quantile spectrum
`freq`	sequence of frequencies
`fit`	object of SAR model
`qser`	matrix or array of quantile series if `y.qser` = `NULL`

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
# compute from time series
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.sar$spec[1,1,sel.f,]))
# compute from quantile series
y.qser <- qser(cbind(y1,y2),tau)
y.sar <- qspec.sar(y.qser=y.qser,tau=tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.sar$spec[1,1,sel.f,]))
```

qspec2qcoh *Quantile Coherence Spectrum*

Description

This function computes quantile coherence spectrum (QCOH) from quantile spectrum of multiple time series.

Usage

```
qspec2qcoh(qspec, k = 1, kk = 2)
```

Arguments

qspec	array of quantile spectrum
k	index of first series (default = 1)
kk	index of second series (default = 2)

Value

matrix of quantile coherence evaluated at Fourier frequencies in (0,0.5)

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qacf <- qacf(cbind(y1,y2),tau)
y.qper.lw <- qspec.lw(y.qacf=y.qacf,M=5)$spec
y.qcoh <- qspec2qcoh(y.qper.lw,k=1,kk=2)
qfa.plot(ff[sel.f],tau,y.qcoh)
```

sar.eq.bootstrap *Bootstrap Simulation of SAR Coefficients for Testing Equality of Granger-Causality in Two Samples*

Description

This function simulates bootstrap samples of selected spline autoregression (SAR) coefficients for testing equality of Granger-causality in two samples based on their SAR models under H0: effect in each sample equals the average effect.

Usage

```
sar.eq.bootstrap(
  y.qser,
  fit,
  fit2,
  index = c(1, 2),
  nsim = 1000,
  method = c("ar", "sar"),
  n.cores = 1,
  mthreads = FALSE,
  seed = 1234567
)
```

Arguments

`y.qser`	matrix or array of QSER from `qser()` or `qspec.sar()$qser`
`fit`	object of SAR model from `qser2sar()` or `qspec.sar()$fit`
`fit2`	object of SAR model for the other sample
`index`	a pair of component indices for multiple time series or a sequence of lags for single time series (default = `c(1,2)`)
`nsim`	number of bootstrap samples (default = 1000)
`method`	method of residual calculation: `"ar"` (default) or `"sar"`
`n.cores`	number of cores for parallel computing (default = 1)
`mthreads`	if `TRUE`, multithread BLAS is enabled when available (default = `FALSE`, required for parallel computing)
`seed`	seed for random sampling (default = `1234567`)

Value

array of simulated bootstrap samples of selected SAR coefficients

Examples

```
y11 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y21 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y12 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y22 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y1.sar <- qspec.sar(cbind(y11,y21),tau=tau,p=1)
y2.sar <- qspec.sar(cbind(y12,y22),tau=tau,p=1)
A1.sim <- sar.eq.bootstrap(y1.sar$qser,y1.sar$fit,y2.sar$fit,index=c(1,2),nsim=5)
A2.sim <- sar.eq.bootstrap(y2.sar$qser,y2.sar$fit,y1.sar$fit,index=c(1,2),nsim=5)
```

sar.eq.test *Wald Test and Confidence Band for Equality of Granger-Causality in Two Samples*

Description

This function computes Wald test and confidence band for equality of Granger-causality in two samples using bootstrap samples generated by `sar.eq.bootstrap()` based on the spline autoregression (SAR) models of quantile series (QSER).

Usage

```
sar.eq.test(A1, A1.sim, A2, A2.sim, sel.lag = NULL, sel.tau = NULL)
```

Arguments

`A1`	matrix of selected SAR coefficients for sample 1
`A1.sim`	simulated bootstrap samples from `sar.eq.bootstrap()` for sample 1
`A2`	matrix of selected SAR coefficients for sample 2
`A2.sim`	simulated bootstrap samples from `sar.eq.bootstrap()` for sample 2
`sel.lag`	indices of time lags for Wald test (default = `NULL`: all lags)
`sel.tau`	indices of quantile levels for Wald test (default = `NULL`: all quantiles)

Value

a list with the following elements:

`test`	list of Wald test result containing `wald` and `p.value`
`D.u`	matrix of upper limits of 95% confidence band for `A1 - A2`
`D.l`	matrix of lower limits of 95% confidence band for `A1 - A2`

Examples

```
y11 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y21 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y12 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y22 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y1.sar <- qspec.sar(cbind(y11,y21),tau=tau,p=1)
y2.sar <- qspec.sar(cbind(y12,y22),tau=tau,p=1)
A1.sim <- sar.eq.bootstrap(y1.sar$qser,y1.sar$fit,y2.sar$fit,index=c(1,2),nsim=5)
A2.sim <- sar.eq.bootstrap(y2.sar$qser,y2.sar$fit,y1.sar$fit,index=c(1,2),nsim=5)
A1 <- sar.gc.coef(y1.sar$fit,index=c(1,2))
A2 <- sar.gc.coef(y2.sar$fit,index=c(1,2))
test <- sar.eq.test(A1,A1.sim,A2,A2.sim,sel.lag=NULL,sel.tau=NULL)
```

`sar.gc.bootstrap`	*Bootstrap Simulation of SAR Coefficients for Granger-Causality Analysis*

Description

This function simulates bootstrap samples of selected spline autoregression (SAR) coefficients for Granger-causality analysis based on the SAR model of quantile series (QSER) under H0: (a) for multiple time series, the second series specified in `index` is not causal for the first series specified in `index`; (b) for single time series, the series is not causal at the lags specified in `index`.

Usage

```
sar.gc.bootstrap(
  y.qser,
  fit,
  index = c(1, 2),
  nsim = 1000,
  method = c("ar", "sar"),
  n.cores = 1,
  mthreads = FALSE,
  seed = 1234567
)
```

Arguments

`y.qser`	matrix or array of QSER from `qser()` or `qspec.sar()$qser`
`fit`	object of SAR model from `qser2sar()` or `qspec.sar()$fit`
`index`	a pair of component indices for multiple time series or a sequence of lags for single time series (default = `c(1,2)`)
`nsim`	number of bootstrap samples (default = 1000)
`method`	method of residual calculation: `"ar"` (default) or `"sar"`
`n.cores`	number of cores for parallel computing (default = 1)
`mthreads`	if `TRUE`, multithread BLAS is enabled when available (default = `FALSE`, required for parallel computing)
`seed`	seed for random sampling (default = `1234567`)

Value

array of simulated bootstrap samples of selected SAR coefficients

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A.sim <- sar.gc.bootstrap(y.sar$qser,y.sar$fit,index=c(1,2),nsim=5)
```

`sar.gc.coef` *Extraction of SAR Coefficients for Granger-Causality Analysis*

Description

This function extracts the spline autoregression (SAR) coefficients from an SAR model for Granger-causality analysis. See `sar.gc.bootstrap` for more details regarding the use of `index`.

Usage

```
sar.gc.coef(fit, index = c(1, 2))
```

Arguments

`fit`	object of SAR model from `qser2sar()` or `qspec.sar()$fit`
`index`	a pair of component indices for multiple time series or a sequence of lags for single time series (default = `c(1,2)`)

Value

matrix of selected SAR coefficients (number of lags by number of quantiles)

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A <- sar.gc.coef(y.sar$fit,index=c(1,2))
```

`sar.gc.test` *Wald Test and Confidence Band for Granger-Causality Analysis*

Description

This function computes Wald test and confidence band for Granger-causality using bootstrap samples generated by `sar.gc.bootstrap()` based the spline autoregression (SAR) model of quantile series (QSER).

Usage

```
sar.gc.test(A, A.sim, sel.lag = NULL, sel.tau = NULL)
```

Arguments

`A`	matrix of selected SAR coefficients
`A.sim`	simulated bootstrap samples from `sar.gc.bootstrap()`
`sel.lag`	indices of time lags for Wald test (default = `NULL`: all lags)
`sel.tau`	indices of quantile levels for Wald test (default = `NULL`: all quantiles)

Value

a list with the following elements:

`test`	list of Wald test result containing `wald` and `p.value`
`A.u`	matrix of upper limits of 95% confidence band of `A`
`A.l`	matrix of lower limits of 95% confidence band of `A`

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A <- sar.gc.coef(y.sar$fit,index=c(1,2))
A.sim <- sar.gc.bootstrap(y.sar$qser,y.sar$fit,index=c(1,2),nsim=5)
y.gc <- sar.gc.test(A,A.sim)
```

`sqr.fit` *Spline Quantile Regression (SQR)*

Description

This function computes spline quantile regression (SQR) solution from response vector and design matrix. It uses the FORTRAN code `rqfnb.f` in the "quantreg" package with the kind permission of Dr. R. Koenker.

Usage

```
sqr.fit(y, X, tau, c0, d = 1, weighted = FALSE, mthreads = FALSE)
```

Arguments

`y`	response vector
`X`	design matrix (`nrow(X) = length(y)`)
`tau`	sequence of quantile levels in (0,1)
`c0`	penalty parameter
`d`	subsampling rate of quantile levels (default = 1)
`weighted`	if `TRUE`, penalty function is weighted (default = `FALSE`)
`mthreads`	if `TRUE`, multithread BLAS is enabled when available (default = `FALSE`, required for parallel computing)

Value

A list with the following elements:

`coefficients`	matrix of regression coefficients
`nit`	number of iterations

`tqr.fit` *Trigonometric Quantile Regression (TQR)*

Description

This function computes trigonometric quantile regression (TQR) for univariate time series at a single frequency.

Usage

```
tqr.fit(y, f0, tau, prepared = TRUE)
```

Arguments

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when f0 = 0.5

Value

object of rq() (coefficients in $coef)

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
plot(tau,fit$coef[1,],type='o',pch=0.75,xlab='QUANTILE LEVEL',ylab='TQR COEF')
```

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