# Package 'lsirm12pl'

March 6, 2025

```
Type Package
Title Latent Space Item Response Model
Version 1.3.4
Date 2025-02-26
Maintainer Gwanghee Kim <musagh08@yonsei.ac.kr>
Description Analysis of dichotomous and continuous response data using latent fac-
     tor by both 1PL LSIRM and 2PL LSIRM as described in Jeon et al. (2021) <doi:10.1007/s11336-
     021-09762-5>. It includes original 1PL LSIRM and 2PL LSIRM provided for binary re-
     sponse data and its extension for continuous response data. Bayesian model selection with spike-
     and-slab prior and method for dealing data with missing value under missing at random, miss-
     ing completely at random are also supported. Various diagnostic plots are available to in-
     spect the latent space and summary of estimated parameters.
License GPL-3
Imports Rcpp (>= 1.0.5), MCMCpack, ggplot2, GPArotation, dplyr (>=
     1.1.4), grDevices, rlang, pROC, coda, spatstat, spatstat.geom,
     spatstat.random, plotly, gridExtra, grid, tidyr, fpc, kernlab,
     plyr, purrr
LinkingTo Rcpp, RcppArmadillo
Encoding UTF-8
RoxygenNote 7.3.2
Suggests knitr, testthat
Depends R (>= 3.1.0)
LazyData true
NeedsCompilation yes
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     Ickhoon Jin [ctb],
     Minjeong Jeon [ctb]
```

Repository CRAN

**Date/Publication** 2025-03-06 10:40:06 UTC

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**BFPT** 

Big Five Personality Test

# **Description**

A dataset containing the result of personality test for 50 questions from 1,000 random sampled people.

# Usage

```
data(BFPT)
```

#### **Format**

A matrix with 1,015,341 rows and 50 columns.

#### **Details**

A dataset collected in 2016-2018 through an interactive on-line personality test, containing the result of personality test for 50 questions. 1,000 people are random sampled from the original dataset containing 1,015,341 people. The scale is labeled as 1=Disagree, 3=Neutral and 5=Agree.

### Source

```
https://www.kaggle.com/tunguz/big-five-personality-test
```

diagnostic

Diagnostic the result of LSIRM.

# **Description**

diagnostic checks the convergence of MCMC for LSIRM parameters using various diagnostic tools, such as trace plots, posterior density distributions, autocorrelation functions (ACF), and Gelman-Rubin-Brooks plots.

```
diagnostic(
  object,
  draw.item = list(beta = c(1), theta = c(1)),
  gelman.diag = FALSE
)
```

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# **Arguments**

object Object of class lsirm.

draw.item List; Each key in the list corresponds to a specific parameters such as "beta",

"theta", "gamma", "alpha", "sigma\_sd", and "zw.dist". The values of the list indicate the indices of these parameters. For the key "zw.dist", the value is a matrix with two columns: the first column represents the indices of respon-

dents, and the second column represents the indices of items.

gelman.diag Logical; If TRUE, the Gelman-Rubin convergence diagnostic will be printed.

Default is FALSE.

#### Value

diagnostic returns plots for checking MCMC convergence for selected parameters.

# **Examples**

```
# Generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# For 1PL LSIRM
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE))
diagnostic(lsirm_result)

# For 2PL LSIRM
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE))
diagnostic(lsirm_result)</pre>
```

gof

Goodness-of-fit LSIRM

# **Description**

gof is goodness-of-fit the latent space of fitted LSIRM.

# Usage

```
gof(object, chain.idx = 1)
```

# **Arguments**

object Object of class lsirm.

chain.idx Numeric; Index of MCMC chain. Default is 1.

# Value

gof returns the boxplot or AUC plot

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# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
lsirm_result <- lsirm(data ~ lsirm1pl())
gof(lsirm_result)</pre>
```

lsirm

Fit a LSIRM ( Latent Space Item Response Model)

# Description

lsirm is used to fit 1PL LSIRM and 2PL LSIRM using Bayesian method as described in Jeon et al. (2021).

# Usage

```
lsirm(formula, ...)
```

# **Arguments**

formula

The form of formula is lsirm(A ~ <term 1> (<term 2>, <term 3> . . .)), where A is binary or continuous item response matrix to be analyzed, <term1> is the model you want to fit and has one of the following values: "lsirm1p1" and "lsirm2p1"., and <term 2>, <term 3>, etc. are each option for the model.

... Additional arguments for the corresponding function.

# **Details**

The descriptions of options for each model, such as <term 2> and <term 3>, are included in lsirm1pl for 1PL LSIRM and lsirm2pl for 2PL LSIRM.

### Value

lsirm returns an object of class list.

See corresponding functions such as lsirm1pl for 1PL LSIRM and lsirm2pl for 2PL LSIRM.

# See Also

```
lsirm1pl for 1PL LSIRM.
lsirm2pl for 2PL LSIRM.
```

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# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
lsirm_result <- lsirm(data~lsirm1pl())
lsirm_result <- lsirm(data~lsirm2pl())</pre>
```

lsirm.formula

Formula function for LSIRM

### **Description**

lsirm.formula is formula object.

# Usage

```
## S3 method for class 'formula'
lsirm(formula, ...)
```

# **Arguments**

formula

The form of formula is  $lsirm(A \sim term 1) < term 2$ , term 3 > ...), where A is binary or continuous item response matrix to be analyzed, term 1 is the model you want to fit and has one of the following values: "lsirm1p1" and "lsirm2p1"., and term 2, term 3, etc., are each option for the model.

... Additional arguments for the corresponding function.

lsirm12pl

lsirm12pl-package

# **Description**

Analysis of dichotomous and continuous response data using latent factor by both 1PL LSIRM and 2PL LSIRM as described in Jeon et al. (2021) <doi:10.1007/s11336-021-09762-5>. It includes original 1PL LSIRM and 2PL LSIRM provided for binary response data and its extension for continuous response data. Bayesian model selection with spike-and-slab prior and method for dealing data with missing value under missing at random, missing completely at random are also supported. Various diagnostic plots are available to inspect the latent space and summary of estimated parameters.

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lsirm1pl

Fit a 1PL LSIRM for binary and continuous item response data

# **Description**

lsirm1pl integrates all functions related to 1PL LSIRM. Various 1PL LSIRM function can be used by setting the spikenslab, fixed\_gamma, and missing\_data arguments.

This function can be used regardless of the data type, providing a unified approach to model fitting.

# Usage

```
lsirm1pl(
  data,
  spikenslab = FALSE,
  fixed_gamma = FALSE,
 missing_data = NA,
  chains = 1,
  multicore = 1,
  seed = NA,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
)
```

# **Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
spikenslab	Logical; specifies whether to use a model selection approach. Default is FALSE.
fixed_gamma	Logical; indicates whether to fix gamma at 1. Default is FALSE.
missing_data	Character; the type of missing data assumed. Options are NA, "mar", or "mcar". Default is NA

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ch	ains	Integer; the number of MCMC chains to run. Default is 1.
mu	lticore	Integer; the number of cores to use for parallel execution. Default is 1.
se	ed	Integer; the seed number for MCMC fitting. Default is NA.
nd	im	Integer; the dimension of the latent space. Default is 2.
ni	ter	Integer; the total number of MCMC iterations to run. Default is 15000.
nb	urn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nt	hin	Integer; the number of MCMC iterations to thin. Default is 5.
np	rint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
ju	mp_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
ju	mp_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
ju	mp_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
ju	mp_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr	_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr	_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr	_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr	_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr	_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
• •	•	Additional arguments for the for various settings. Refer to the functions in the Details.

#### **Details**

Additional arguments and return values for each function are documented in the respective function's description.

- \* For LSIRM with data included missing value are detailed in lsirm1pl\_mar and lsirm1pl\_mcar.
- \* For LSIRM using the spike-and-slab model selection approach are detailed in <a href="lsirm1pl\_ss">lsirm1pl\_ss</a>.
- \* For continuous version of LSIRM are detailed in <a href="mailto:lsirm1pl\_normal\_o">lsirm1pl\_normal\_o</a>.

For 1PL LSIRM with binary item response data, the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$logit(P(Y_{i,i} = 1 | \theta_i, \beta_i, \gamma, z_i, w_i)) = \theta_i + \beta_i - \gamma ||z_i - w_i||$$

For 1PL LSIRM with continuous item response data, the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ .

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#### Value

lsirm1pl returns an object of list. The basic return list containing the following components:

data A data frame or matrix containing the variables used in the model.

bic A numeric value representing the Bayesian Information Criterion (BIC).

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter.
w\_estimate Posterior estimates of the w parameter.
beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

W Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

... Additional return values for various settings. Refer to the functions in the De-

tails.

# Note

If both spikenslab and fixed\_gamma are set TRUE, it returns error because both are related to gamma.

#### See Also

The LSIRM for 1PL LSIRM for binary item response data as following:

lsirm1pl\_o, lsirm1pl\_fixed\_gamma, lsirm1pl\_mar,lsirm1pl\_mcar, lsirm1pl\_fixed\_gamma\_mar,
lsirm1pl\_fixed\_gamma\_mcar, lsirm1pl\_ss, lsirm1pl\_mar\_ss, and lsirm1pl\_mcar\_ss

The LSIRM for 1PL LSIRM for continuous item response data as following:

lsirm1pl\_normal\_o, lsirm1pl\_normal\_fixed\_gamma, lsirm1pl\_normal\_mar, lsirm1pl\_normal\_mcar,lsirm1pl\_norm
lsirm1pl\_normal\_fixed\_gamma\_mcar, lsirm1pl\_normal\_ss, lsirm1pl\_normal\_mar\_ss, lsirm1pl\_normal\_mcar\_ss

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
lsirm_result <- lsirm1pl(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data~lsirm1pl())</pre>
```

# **Description**

lsirm1pl\_fixed\_gamma is used to fit 1PL LSIRM with gamma fixed to 1. lsirm1pl\_fixed\_gamma factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm1pl_fixed_gamma(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
 pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_a_theta = 0.001,
 pr_b_team = 0.001,
  verbose = FALSE
)
```

# **Arguments**

data Matrix; a binary or continuous item response matrix for analysis. Each row rep-

resents a respondent, and each column contains responses to the corresponding

item.

ndim Integer; the dimension of the latent space. Default is 2.

niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is $2500$ .
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .

# **Details**

verbose

lsirm1pl\_fixed\_gamma models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space:

Logical; If TRUE, MCMC samples are printed for each nprint. default value

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, z_j, w_i)) = \theta_j + \beta_i - ||z_j - w_i||$$

# Value

lsirm1pl\_fixed\_gamma returns an object of list containing the following components:

data Data frame or matrix containing	g the variables in the model.
--------------------------------------	-------------------------------

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

is FALSE

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter.

w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
Z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
W	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

lsirm_result <- lsirm1pl_fixed_gamma(data)

# The code following can achieve the same result.

lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE))</pre>
```

lsirm1pl\_fixed\_gamma\_mar

1PL LSIRM fixing gamma to 1 for missing at random data.

# Description

lsirm1pl\_fixed\_gamma\_mar is used to fit LSIRM with gamma fixed to 1 in incomplete data assumed to be missing at random. lsirm1pl\_fixed\_gamma\_mar factorizes item response matrix into columnwise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

```
lsirm1pl_fixed_gamma_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
```

```
jump_beta = 0.4,
jump_theta = 1,
jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
missing.val = 99,
verbose = FALSE
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE.

#### **Details**

lsirm1pl\_fixed\_gamma\_mar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space:

$$logit(P(Y_{i,i} = 1 | \theta_i, \beta_i, z_i, w_i)) = \theta_i + \beta_i - ||z_i - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References

#### Value

lsirm1pl\_fixed\_gamma\_mar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.

theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter. w\_estimate Posterior estimates of the w parameter.

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

# References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_fixed_gamma_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE, missing_data = "mar", missing.val = 99))</pre>
```

lsirm1pl\_fixed\_gamma\_mcar

1PL LSIRM fixing gamma to 1 for missing completely at random data.

# **Description**

lsirm1pl\_fixed\_gamma\_mcar is used to fit LSIRM with gamma fixed to 1 in incomplete data assumed to be missing completely at random. lsirm1pl\_fixed\_gamma\_mcar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

```
lsirm1pl_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
```

```
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
missing.val = 99,
verbose = FALSE
)
```

# **Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE

# **Details**

lsirm1pl\_fixed\_gamma\_mcar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space:

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, z_j, w_i)) = \theta_j + \beta_i - ||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

#### Value

lsirm1pl\_fixed\_gamma\_mcar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter.
 w\_estimate Posterior estimates of the w parameter.
 beta Posterior samples of the beta parameter.
 theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.

#### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)
# make missing value with missing indicator matrix
data[missing_mat==1] <- 99</pre>
```

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```
lsirm_result <- lsirm1pl_fixed_gamma_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE, missing_data = "mcar", missing.val = 99))</pre>
```

lsirm1pl\_mar

1PL LSIRM for missing at random data.

# Description

lsirm1pl\_mar is used to fit 1PL LSIRM in incomplete data assumed to be missing at random. lsirm1pl\_mar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm1pl_mar(
  data,
 ndim = 2,
 niter = 15000,
 nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
 pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
 pr_a_theta = 0.001,
 pr_b_t = 0.001,
 missing.val = 99,
  verbose = FALSE
)
```

# **Arguments**

data

Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.

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ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is $2500$ .
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE.

# **Details**

lsirm1pl\_mar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

#### Value

lsirm1pl\_mar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

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mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.
gamma Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

accept\_gamma Acceptance ratio for the gamma parameter.

# References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_mar(data)</pre>
```

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```
# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE,
missing_data ='mar', missing.val = 99))</pre>
```

lsirm1pl\_mar\_ss

1PL LSIRM with model selection approach for missing at random data.

# **Description**

lsirm1pl\_mar\_ss is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors in incomplete data assumed to be missing at random. lsirm1pl\_mar\_ss factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

```
lsirm1pl_mar_ss(
  data,
 ndim = 2,
 niter = 15000.
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_spike_mean = -3,
  pr_spike_sd = 1,
  pr_slab_mean = 0.5,
 pr_slab_sd = 1,
 pr_a_theta = 0.001,
  pr_b_t = 0.001,
  pr_xi_a = 1,
 pr_xi_b = 1,
 missing.val = 99,
  verbose = FALSE
)
```

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# Arguments

4-4-	Matrice a him are a section and its answer and the form and the first
data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

# **Details**

lsirm1pl\_mar\_ss models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

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Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm1pl\_mar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

#### Value

lsirm1pl\_mar\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

pi Posterior samples of phi which is indicator of spike and slab prior. If phi is 1,

log gamma follows the slab prior, otherwise follows the spike prior.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.
accept\_gamma Acceptance ratio for the gamma parameter.

pi\_estimate Posterior estimation of phi. inclusion probability of gamma. if estimation of phi

is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space

model with gamma > 0.

imp\_estimate Probability of imputating a missing value with 1.

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#### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

#### **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_mar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE, missing_data = 'mar', missing = 99))</pre>
```

lsirm1pl\_mcar

1PL LSIRM for missing completely at random data.

# **Description**

lsirm1pl\_mcar is used to fit 1PL LSIRM in incomplete data assumed to be missing completely at random. lsirm1pl\_mcar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

```
lsirm1pl_mcar(
   data,
   ndim = 2,
   niter = 15000,
   nburn = 2500,
   nthin = 5,
   nprint = 500,
   jump_beta = 0.4,
   jump_theta = 1,
   jump_gamma = 0.025,
   jump_z = 0.5,
   jump_w = 0.5,
```

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```
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_mean_gamma = 0.5,
pr_sd_gamma = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
missing.val = 99,
verbose = FALSE
)
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

#### **Details**

Isirm1p1\_mcar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

#### Value

lsirm1pl\_mcar returns an object of list containing the following components:

data A data frame or matrix containing the variables used in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.
accept\_gamma Acceptance ratio for the gamma parameter.

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#### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons

#### **Examples**

lsirm1pl\_mcar\_ss

1PL LSIRM with model selection approach for missing completely at random data.

# **Description**

lsirm1pl\_mcar\_ss is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors in incomplete data assumed to be missing completely at random. lsirm1pl\_mcar\_ss factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

```
lsirm1pl_mcar_ss(
   data,
   ndim = 2,
   niter = 15000,
   nburn = 2500,
   nthin = 5,
   nprint = 500,
   jump_beta = 0.4,
   jump_theta = 1,
   jump_gamma = 1,
   jump_z = 0.5,
```

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```
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 1,
pr_xi_b = 1,
missing.val = 99,
verbose = FALSE
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

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pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

#### **Details**

lsirm1pl\_mcar\_ss models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References. lsirm1pl\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm1pl\_mcar\_ss returns an object of list containing the following components:

data	Data frame o	r matrix co	ntaining the	variables in the mode	4

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

gamma Posterior samples of the gamma parameter.

theta_sd	Posterior samples of the standard deviation of theta.
Z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
W	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.

#### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

# **Examples**

lsirm1pl\_normal\_fixed\_gamma

1PL LSIRM fixing gamma to 1 with normal likelihood

# Description

lsirm1pl\_normal\_fixed\_gamma is used to fit 1PL LSIRM for continuous variable with gamma fixed to 1. lsirm1pl\_normal\_fixed\_gamma factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm1pl_normal_fixed_gamma(
  data,
  ndim = 2,
 niter = 15000,
 nburn = 2500,
 nthin = 5,
 nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
 pr_mean_beta = 0,
 pr_sd_beta = 1,
  pr_mean_theta = 0,
 pr_a_theta = 0.001,
 pr_b_teta = 0.001,
 pr_a_{eps} = 0.001,
 pr_b_{eps} = 0.001,
  verbose = FALSE
)
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE.

#### **Details**

lsirm1pl\_normal\_fixed\_gamma models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space:

$$Y_{j,i} = \theta_j + \beta_i - ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ .

#### Value

lsirm1pl\_normal\_fixed\_gamma returns an object of list containing the following components:

data A data frame or matrix containing the variables used in the model. A numeric value representing the Bayesian Information Criterion (BIC). bic mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning intervals. map\_inf The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs. beta\_estimate Posterior estimates of the beta parameter. theta\_estimate Posterior estimates of the theta parameter. sigma\_theta\_estimate Posterior estimates of the standard deviation of theta. Posterior estimates of the z parameter. z\_estimate w estimate Posterior estimates of the w parameter.

Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the standard deviction.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.

```
accept_z Acceptance ratio for the z parameter.

accept_w Acceptance ratio for the w parameter.

sigma_estimate Posterior estimates of the standard deviation.

sigma Posterior samples of the standard deviation.
```

# **Examples**

```
lsirm1pl_normal_fixed_gamma_mar
```

1PL LSIRM fixing gamma to 1 with normal likelihood for missing at random data.

# Description

lsirm1pl\_normal\_fixed\_gamma\_mar is used to fit 1PL LSIRM for continuous variable with gamma fixed to 1 in incomplete data assumed to be missing at random.

lsirm1pl\_normal\_fixed\_gamma\_mar factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

```
lsirm1pl_normal_fixed_gamma_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
```

```
jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_b_eps = 0.001,
missing.val = 99,
verbose = FALSE
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

#### **Details**

lsirm1pl\_normal\_fixed\_gamma\_mar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_i$  of respondent j in the shared metric space:

$$Y_i, i = \theta_i + \beta_i - ||z_i - w_i|| + e_{ii}$$

where the error  $e_j i \ N(0, \sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

#### Value

lsirm1pl\_normal\_fixed\_gamma\_mar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter. w\_estimate Posterior estimates of the w parameter.

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

W Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.

sigma\_estimate Posterior estimates of the standard deviation.
sigma Posterior samples of the standard deviation.

# **Examples**

lsirm1pl\_normal\_fixed\_gamma\_mcar

1PL LSIRM fixing gamma to 1 with normal likelihood for missing completely at random data.

# Description

lsirm1pl\_normal\_fixed\_gamma\_mcar is used to fit 1PL LSIRM for continuous variable with gamma fixed to 1 in incomplete data assumed to be missing completely at random.

lsirm1pl\_normal\_fixed\_gamma\_mcar factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

```
lsirm1pl_normal_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_a_theta = 0.001,
```

```
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
missing.val = 99,
verbose = FALSE
)
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## **Details**

lsirm1pl\_normal\_fixed\_gamma\_mcar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space:

$$Y_{j,i} = \theta_j + \beta_i - ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

#### Value

lsirm1pl\_normal\_fixed\_gamma\_mcar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter.
w\_estimate Posterior estimates of the w parameter.
beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

sigma\_estimate Posterior estimates of the standard deviation.

sigma Posterior samples of the standard deviation.

# Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)
# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)</pre>
```

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```
# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_fixed_gamma_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE, missing_data = "mcar", missing.val = 99))</pre>
```

lsirm1pl\_normal\_mar

1PL LSIRM with normal likelihood for missing at random data.

### **Description**

lsirm1pl\_normal\_mar is used to fit LSIRM for continuous variable with 1pl in incomplete data assumed to be missing at random. lsirm1pl\_normal\_mar factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm1pl_normal_mar(
  data,
  ndim = 2,
 niter = 15000.
 nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
 pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
 pr_sd_gamma = 1,
 pr_a_theta = 0.001,
  pr_b_t = 0.001,
 pr_a_{eps} = 0.001,
 pr_b_{eps} = 0.001,
 missing.val = 99,
  verbose = FALSE
)
```

### **Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm1pl\_normal\_mar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_j, i = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{ji}$$

where the error  $e_{ji} \sim N(0, \sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

#### Value

lsirm1pl\_normal\_mar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.
gamma Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

accept\_gamma Acceptance ratio for the gamma parameter.

sigma\_estimate Posterior estimates of the standard deviation.

Posterior samples of the standard deviation.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

# **Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE, missing_data = 'mar', missing.val = 99))</pre>
```

lsirm1pl\_normal\_mar\_ss

1PL LSIRM with normal likelihood and model selection approach for missing at random data.

# **Description**

lsirm1pl\_normal\_mar\_ss is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors for continuous variable with 1pl in incomplete data assumed to be missing at random. lsirm1pl\_normal\_mar\_ss factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_normal_mar_ss(
    data,
    ndim = 2,
    niter = 15000,
    nburn = 2500,
    nthin = 5,
    nprint = 500,
    jump_beta = 0.4,
    jump_theta = 1,
    jump_gamma = 1,
    jump_z = 0.5,
    jump_w = 0.5,
    pr_mean_beta = 0,
    pr_sd_beta = 1,
    pr_mean_theta = 0,
```

```
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
pr_xi_a = 0.001,
pr_xi_b = 0.001,
missing.val = 99,
verbose = FALSE
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm1pl\_normal\_mar\_ss models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0,\sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm1pl\_normal\_mar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

# Value

w\_estimate

beta

lsirm1pl\_normal\_mar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_est	imate
	Posterior estimates of the standard deviation of theta.
<pre>gamma_estimate</pre>	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.

Posterior estimates of the w parameter. Posterior samples of the beta parameter.

theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
Z	Posterior samples of the $z$ parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
W	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than $0.5$ , choose Rasch model with gamma = $0$ , otherwise latent space model with gamma > $0$ .
<pre>imp_estimate</pre>	Probability of imputating a missing value with 1.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.

### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

# **Examples**

### **Description**

lsirm1pl\_normal\_mcar is used to fit LSIRM with 1pl in incomplete data assumed to be missing completely at random. lsirm1pl\_normal\_mcar factorizes continuous item response matrix into columnwise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_normal_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
 pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
  pr_a_{eps} = 0.001,
 pr_b_eps = 0.001,
 missing.val = 99,
  verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding
	item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.

nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

# **Details**

lsirm1pl\_normal\_mcar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0,\sigma^2)$  Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing at random assumption and data augmentation, see References.

#### Value

lsirm1pl\_normal\_mcar returns an object of list containing the following components:

data A data frame or matrix containing the variables used in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.

accept\_gamma Acceptance ratio for the gamma parameter.
sigma\_estimate Posterior estimates of the standard deviation.
sigma Posterior samples of the standard deviation.

#### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## **Examples**

lsirm1pl\_normal\_mcar\_ss

1PL LSIRM with normal likelihood and model selection approach for missing completely at random data.

# **Description**

lsirm1pl\_normal\_mcar\_ss is used to fit LSIRM with model selection approach based on spike-and-slab priors for continuous variable with 1pl in incomplete data assumed to be missing completely at random. lsirm1pl\_normal\_mcar\_ss factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_normal_mcar_ss(
    data,
    ndim = 2,
    niter = 15000,
    nburn = 2500,
    nthin = 5,
    nprint = 500,
    jump_beta = 0.4,
    jump_theta = 1,
    jump_gamma = 1,
    jump_z = 0.5,
    jump_w = 0.5,
    pr_mean_beta = 0,
    pr_sd_beta = 1,
    pr_mean_theta = 0,
```

```
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
pr_xi_a = 0.001,
pr_xi_b = 0.001,
missing.val = 99,
verbose = FALSE
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm1pl\_normal\_mcar\_ss models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0,\sigma^2)$ . Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing at random assumption and data augmentation, see References. lsirm1pl\_normal\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

# Value

w\_estimate

beta

lsirm1pl\_normal\_mcar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_est	imate
	Posterior estimates of the standard deviation of theta.
<pre>gamma_estimate</pre>	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.

Posterior estimates of the w parameter.

Posterior samples of the beta parameter.

theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the $z$ parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
W	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.

### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

# **Examples**

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lsirm1pl\_normal\_o

1PL LSIRM with normal likelihood.

# **Description**

lsirm1pl\_normal\_o is used to fit LSIRM for continuous variable with 1pl. lsirm1pl\_normal\_o factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm1pl_normal_o(
  data,
 ndim = 2,
 niter = 15000,
 nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
 pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_mean_gamma = 0.5,
 pr_sd_gamma = 1,
 pr_a_theta = 0.001,
  pr_b_t = 0.001,
 pr_a_{eps} = 0.001,
 pr_b_{eps} = 0.001,
  verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.

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nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm1pl\_normal\_o models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ .

#### Value

lsirm1pl\_normal\_o returns an object of list containing the following components:

data Data frame or matrix containing the variables used in the model.

bic A numeric value representing the Bayesian Information Criterion (BIC).

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.

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theta\_estimate Posterior estimates of the theta parameter. sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.
accept\_gamma Acceptance ratio for the gamma parameter.
sigma\_estimate Posterior estimates of the standard deviation.
Posterior samples of the standard deviation.

#### **Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

lsirm_result <- lsirm1pl_normal_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE))</pre>
```

lsirm1pl\_normal\_ss

1PL LSIRM with normal likelihood and model selection approach.

# Description

lsirm1pl\_normal\_ss is used to fit LSIRM with model selection approach based on spike-and-slab priors for continuous variable with 1pl. LSIRM factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

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# Usage

```
lsirm1pl_normal_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
 pr_spike_mean = -3,
 pr_spike_sd = 1,
  pr_slab_mean = 0.5,
 pr_slab_sd = 1,
 pr_a_theta = 0.001,
 pr_b_t = 0.001,
  pr_a_{eps} = 0.001,
  pr_b_{eps} = 0.001,
 pr_xi_a = 0.001,
 pr_xi_b = 0.001,
  verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.

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jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; mean of spike prior for log gamma default value is -3.
pr_spike_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_slab_mean	Numeric; mean of spike prior for log gamma default value is 0.5.
pr_slab_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; scale parameter of inverse gamma prior for variance of data likelihood. Default is $0.001$ .
pr_xi_a	Numeric; first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

#### **Details**

lsirm1pl\_normal\_ss models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . lsrm1pl\_noraml\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

# Value

lsirm1pl\_normal\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

bic Numeric value with the corresponding BIC.

mcmc\_inf number of mcmc iteration, burn-in periods, and thinning intervals.

map\_inf value of log maximum a posterior and iteration number which have log maxi-

mum a posterior.

beta\_estimate posterior estimation of beta. theta\_estimate posterior estimation of theta. 58 Isirm1pl\_normal\_ss

sigma\_theta\_estimate

posterior estimation of standard deviation of theta.

sigma\_estimate posterior estimation of standard deviation.

gamma\_estimate posterior estimation of gamma.

 $z_{\text{estimate}}$  posterior estimation of z.  $w_{\text{estimate}}$  posterior estimation of w.

pi\_estimate posterior estimation of phi. inclusion probability of gamma. if estimation of phi

is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space

model with gamma > 0.

beta posterior samples of beta. theta posterior samples of theta.

theta\_sd posterior samples of standard deviation of theta.

sigma posterior samples of standard deviation.

gamma posterior samples of gamma.

z posterior samples of z. The output is 3-dimensional matrix with last axis repre-

sent the dimension of latent space.

w posterior samples of w. The output is 3-dimensional matrix with last axis repre-

sent the dimension of latent space.

pi posterior samples of phi which is indicator of spike and slab prior. If phi is 1,

log gamma follows the slab prior, otherwise follows the spike prior.

accept\_beta accept ratio of beta.

accept\_theta accept ratio of theta.

accept\_w accept ratio of w.

accept\_z accept ratio of z.

accept\_gamma accept ratio of gamma.

#### References

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies (Vol. 33). The Annals of Statistics

## **Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

lsirm_result <- lsirm1pl_normal_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE))</pre>
```

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lsirm1pl\_o *1PL LSIRM*.

# **Description**

lsirm1pl\_o is used to fit 1PL LSIRM. lsirm1pl\_o factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm1pl_o(
 data,
 ndim = 2,
 niter = 15000,
 nburn = 2500,
 nthin = 5,
 nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_mean_gamma = 0.5,
 pr_sd_gamma = 1,
 pr_a_theta = 0.001,
 pr_b_t = 0.001,
  verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is $2500$ .
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.

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jump\_beta Numeric; the jumping rule for the beta proposal density. Default is 0.4. Numeric; the jumping rule for the theta proposal density. Default is 1.0. jump\_theta Numeric; the jumping rule for the gamma proposal density. Default is 0.025. jump\_gamma jump\_z Numeric; the jumping rule for the z proposal density. Default is 0.5. Numeric; the jumping rule for the w proposal density. Default is 0.5. jump\_w Numeric; the mean of the normal prior for beta. Default is 0. pr\_mean\_beta pr\_sd\_beta Numeric; the standard deviation of the normal prior for beta. Default is 1.0. Numeric; the mean of the normal prior for theta. Default is 0. pr\_mean\_theta Numeric; mean of log normal prior for gamma. Default is 0.5. pr\_mean\_gamma pr\_sd\_gamma Numeric; standard deviation of log normal prior for gamma. Default is 1.0. Numeric; the shape parameter of the inverse gamma prior for the variance of pr\_a\_theta theta. Default is 0.001. pr\_b\_theta Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001. verbose Logical; If TRUE, MCMC samples are printed for each nprint. Default is

#### **Details**

lsirm1pl\_o models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$logit(P(Y_{i,i} = 1 | \theta_i, \beta_i, \gamma, z_i, w_i)) = \theta_i + \beta_i - \gamma ||z_i - w_i||$$

#### Value

w\_estimate

beta

lsirm1pl\_o returns an object of list containing the following components:

Posterior estimates of the w parameter.

Posterior samples of the beta parameter.

FALSE.

Data frame or matrix containing the variables used in the model. data bic A numeric value representing the Bayesian Information Criterion (BIC). mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning intervals. The log maximum a posteriori (MAP) value and the iteration number at which map\_inf this MAP value occurs. Posterior estimates of the beta parameter. beta\_estimate theta\_estimate Posterior estimates of the theta parameter. sigma\_theta\_estimate Posterior estimates of the standard deviation of theta. gamma\_estimate Posterior estimates of gamma parameter. Posterior estimates of the z parameter. z\_estimate

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theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
Z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
W	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

lsirm_result <- lsirm1pl_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE))</pre>
```

lsirm1pl\_ss

1PL LSIRM with model selection approach.

# **Description**

lsirm1pl\_ss is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors. LSIRM factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
```

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```
jump_gamma = 1,
jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 1,
pr_xi_b = 1,
verbose = FALSE
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .

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pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

# **Details**

lsirm1pl\_ss models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$ of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

lsirm1pl\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

#### Value

gamma

W

Posterior samples of the gamma parameter.

lsirm1pl_ss returns an object of list containing the following components:		
data	Data frame or matrix containing the variables used in the model.	
bic	A numeric value representing the Bayesian Information Criterion (BIC).	
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.	
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.	
beta_estimate	Posterior estimates of the beta parameter.	
theta_estimate	Posterior estimates of the theta parameter.	
sigma_theta_estimate		
	Posterior estimates of the standard deviation of theta.	
<pre>gamma_estimate</pre>	Posterior estimates of gamma parameter.	
z_estimate	Posterior estimates of the z parameter.	
w_estimate	Posterior estimates of the w parameter.	
beta	Posterior samples of the beta parameter.	
theta	Posterior samples of the theta parameter.	
theta_sd	Posterior samples of the standard deviation of theta.	

where the last axis denotes the dimension of the latent space.

where the last axis denotes the dimension of the latent space.

Posterior samples of the z parameter, represented as a 3-dimensional matrix

Posterior samples of the w parameter, represented as a 3-dimensional matrix

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accept\_beta Acceptance ratio for the beta parameter. Acceptance ratio for the theta parameter. accept\_theta accept\_z Acceptance ratio for the z parameter. accept\_w Acceptance ratio for the w parameter. accept\_gamma Acceptance ratio for the gamma parameter. Posterior estimation of phi. inclusion probability of gamma. if estimation of phi pi\_estimate is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0. Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, рi log gamma follows the slab prior, otherwise follows the spike prior.

#### References

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies (Vol. 33). The Annals of Statistics

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

lsirm_result <- lsirm1pl_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE))</pre>
```

lsirm2pl

Fit a 2pl LSIRM for binary and continuous item resopnse data

#### **Description**

lsirm2pl integrates all functions related to 2PL LSIRM. Various 2PL LSIRM function can be used by setting the spikenslab, fixed\_gamma, and missing\_data arguments.

This function can be used regardless of the data type, providing a unified approach to model fitting.

# Usage

```
lsirm2pl(
  data,
  spikenslab = FALSE,
  fixed_gamma = FALSE,
  missing_data = NA,
  chains = 1,
  multicore = 1,
  seed = NA,
```

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```
ndim = 2,
 niter = 15000,
 nburn = 2500,
 nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
 pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_a_theta = 0.001,
 pr_b_teta = 0.001,
 pr_mean_alpha = 0.5,
 pr_sd_alpha = 1,
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
spikenslab	Logical; specifies whether to use a model selection approach. Default is FALSE.
fixed_gamma	Logical; indicates whether to fix gamma at 1. Default is FALSE.
missing_data	Character; the type of missing data assumed. Options are NA, "mar", or "mcar". Default is NA.
chains	Integer; the number of MCMC chains to run. Default is 1.
multicore	Integer; the number of cores to use for parallel execution. Default is 1.
seed	Integer; the seed number for MCMC fitting. Default is NA.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is $2500$ .
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.

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pr\_mean\_beta Numeric; the mean of the normal prior for beta. Default is 0. Numeric; the standard deviation of the normal prior for beta. Default is 1.0. pr\_sd\_beta Numeric; the mean of the normal prior for theta. Default is 0. pr\_mean\_theta pr\_a\_theta Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001. Numeric; the scale parameter of the inverse gamma prior for the variance of pr\_b\_theta theta. Default is 0.001. pr\_mean\_alpha Numeric; the mean of the log normal prior for alpha. Default is 0.5. pr\_sd\_alpha Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0. Additional arguments for the for various settings. Refer to the functions in the

#### **Details**

Additional arguments and return values for each function are documented in the respective function's description.

- \* For 2PL LSIRM with data included missing value are detailed in <a href="lsirm2pl\_mar">lsirm2pl\_mar</a> and <
- \* For 2PL LSIRM using the spike-and-slab model selection approach are detailed in lsirm2pl\_ss.
- \* For continuous version of 2PL LSIRM are detailed in <a href="mailto:lsirm2pl\_normal\_o">lsirm2pl\_normal\_o</a>.

Details.

For 2PL LSIRM with binary item response data, the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

For 2PL LSIRM with continuous item response data, the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_i$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{i,i} \sim N(0, \sigma^2)$ 

### Value

lsirm2pl returns an object of list. The basic return list containing the following components:

data A data frame or matrix containing the variables used in the model.

bic A numeric value representing the Bayesian Information Criterion (BIC).

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

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map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

alpha\_estimate posterior estimates of alpha parameter..

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

alpha Posterior samples of the alpha parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.
accept\_alpha Acceptance ratio for the alpha parameter.

... Additional return values for various settings. Refer to the functions in the De-

tails.

#### Note

If both spikenslab and fixed\_gamma are set TRUE, it returns error because both are related to gamma.

#### See Also

The 2PL LSIRM for binary item response data as following:

lsirm2pl\_o, lsirm2pl\_fixed\_gamma, lsirm2pl\_mar,lsirm2pl\_mcar, lsirm2pl\_fixed\_gamma\_mar,
lsirm2pl\_fixed\_gamma\_mcar, lsirm2pl\_ss, lsirm2pl\_mar\_ss, and lsirm2pl\_mcar\_ss

The 2PL LSIRM for continuous item response data as following:

lsirm2pl\_normal\_o, lsirm2pl\_normal\_fixed\_gamma, lsirm2pl\_normal\_mar, lsirm2pl\_normal\_mcar,lsirm1pl\_norm
lsirm2pl\_normal\_fixed\_gamma\_mcar, lsirm2pl\_normal\_ss, lsirm2pl\_normal\_mar\_ss, lsirm2pl\_normal\_mcar\_ss

### **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
lsirm_result <- lsirm2pl(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data~lsirm2pl())</pre>
```

lsirm2pl\_fixed\_gamma 2PL LSIRM fixing gamma to 1.

# Description

lsirm2pl\_fixed\_gamma is used to fit 2PL LSIRM fixing gamma to 1. lsirm2pl\_fixed\_gamma factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm2pl_fixed_gamma(
  data,
  ndim = 2,
 niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
 pr_b_t = 0.001,
  verbose = FALSE
)
```

### **Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

# **Details**

lsirm2pl\_fixed\_gamma models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - ||z_j - w_i||$$

# Value

lsirm2pl\_fixed\_gamma returns an object of list containing the following components: lsirm1pl\_fixed\_gamma returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter.
 w\_estimate Posterior estimates of the w parameter.
 beta Posterior samples of the beta parameter.
 theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

alpha\_estimate Posterior estimates of the alpha parameter.

alpha Posterior estimates of the alpha parameter.

accept\_alpha Acceptance ratio for the alpha parameter.

### **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

lsirm_result <- lsirm2pl_fixed_gamma(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE))</pre>
```

```
lsirm2pl_fixed_gamma_mar
```

2PL LSIRM fixing gamma to 1 for missing at random data.

# **Description**

lsirm2pl\_fixed\_gamma\_mar is used to fit 2PL LSIRM fixing gamma to 1 in incomplete data assumed to be missing at random. lsirm2pl\_fixed\_gamma\_mar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1pl model, 2pl model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm2pl_fixed_gamma_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
 pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
 missing.val = 99,
  verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.

nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_fixed\_gamma\_mar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

#### Value

 ${\tt lsirm2pl\_fixed\_gamma\_mar}\ \ {\tt returns}\ \ {\tt an}\ \ {\tt object}\ \ {\tt of}\ \ {\tt list}\ \ {\tt containing}\ \ {\tt the}\ \ {\tt following}\ \ {\tt components};$ 

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter. w\_estimate Posterior estimates of the w parameter.

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

alpha\_estimate Posterior estimates of the alpha parameter.

alpha Posterior estimates of the alpha parameter.

accept\_alpha Acceptance ratio for the alpha parameter.

### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_fixed_gamma_mar(data)</pre>
```

lsirm2pl\_fixed\_gamma\_mcar

2PL LSIRM fixing gamma to 1 for missing completely at random data.

## Description

lsirm2pl\_fixed\_gamma\_mcar is used to fit 2PL LSIRM fixing gamma to 1 in incomplete data assumed to be missing completely at random. lsirm2pl\_fixed\_gamma\_mcar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1pl model, 2pl model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
 missing.val = 99,
  verbose = FALSE
)
```

### **Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_fixed\_gamma\_mcar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - ||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

### Value

lsirm2pl\_fixed\_gamma\_mar returns an object of list containing the following components:

Data frame or matrix containing the variables in the model. data

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter. theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

Posterior estimates of the z parameter. z\_estimate Posterior estimates of the w parameter. w\_estimate beta Posterior samples of the beta parameter. theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

Posterior samples of the z parameter, represented as a 3-dimensional matrix 7

where the last axis denotes the dimension of the latent space.

Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter. accept\_theta Acceptance ratio for the theta parameter. Acceptance ratio for the z parameter. accept\_z Acceptance ratio for the w parameter. accept\_w alpha\_estimate Posterior estimates of the alpha parameter. alpha Posterior estimates of the alpha parameter.

accept\_alpha Acceptance ratio for the alpha parameter.

### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example item response matrix
         <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
# generate example missing indicator matrix
missing_mat
                <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)</pre>
```

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lsirm2pl\_mar

2PL LSIRM for missing at random data.

## **Description**

lsirm2pl\_mar is used to fit 2PL LSIRM in incomplete data assumed to be missing at random. lsirm2pl\_mar factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### **Usage**

```
lsirm2pl_mar(
  data,
  ndim = 2,
 niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
 pr_a_theta = 0.001,
  pr_b_t = 0.001,
 missing.val = 99,
```

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```
verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

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### **Details**

lsirm2pl\_mar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

### Value

lsirm2pl\_mar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.

theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.
gamma Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

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```
accept_w Acceptance ratio for the w parameter.
accept_gamma Acceptance ratio for the gamma parameter.
alpha_estimate Posterior estimates of the alpha parameter.
alpha Posterior estimates of the alpha parameter.
accept_alpha Acceptance ratio for the alpha parameter.
```

#### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE, missing_data = "mar"))</pre>
```

lsirm2pl\_mar\_ss

2PL LSIRM with model selection approach for missing at random data.

### **Description**

lsirm2pl\_mar\_ss is used to fit 2PL LSIRM based on spike-and-slab priors in incomplete data assumed to be missing at random. lsirm2pl\_mar\_ss factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

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## Usage

```
lsirm2pl_mar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump\_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_spike_mean = -3,
  pr_spike_sd = 1,
  pr_slab_mean = 0.5,
  pr_slab_sd = 1,
 pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
 pr_b_t = 0.001,
 pr_xi_a = 1,
 pr_xi_b = 1,
 missing.val = 99,
  verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.

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jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_mar\_ss models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm2pl\_mar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm2pl\_mar\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

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missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

pi Posterior samples of phi which is indicator of spike and slab prior. If phi is 1,

log gamma follows the slab prior, otherwise follows the spike prior.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.
accept\_w Acceptance ratio for the w parameter.
accept\_gamma Acceptance ratio for the gamma parameter.

pi\_estimate Posterior estimation of phi. inclusion probability of gamma. if estimation of phi

is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space

model with gamma > 0.

imp\_estimate Probability of imputating a missing value with 1.

alpha\_estimate Posterior estimates of the alpha parameter.
alpha Posterior estimates of the alpha parameter.
accept\_alpha Acceptance ratio for the alpha parameter.

### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

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### **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE, missing_data = "mar"))</pre>
```

lsirm2pl\_mcar

2PL LSIRM for missing completely at random data.

## Description

lsirm2pl\_mcar is used to fit 2PL LSIRM in incomplete data assumed to be missing completely at random. lsirm2pl\_mcar factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
```

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```
pr_mean_gamma = 0.5,
pr_sd_gamma = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
missing.val = 99,
verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

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### **Details**

lsirm2pl\_mcar models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

### Value

lsirm2pl\_mar returns an object of list containing the following components:

data A data frame or matrix containing the variables used in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter. accept\_theta Acceptance ratio for the theta parameter. accept\_z Acceptance ratio for the z parameter. Acceptance ratio for the w parameter. accept\_w accept\_gamma Acceptance ratio for the gamma parameter. alpha\_estimate Posterior estimates of the alpha parameter. alpha Posterior estimates of the alpha parameter. accept\_alpha Acceptance ratio for the alpha parameter.

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### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons

### **Examples**

```
# generate example item response matrix
data     <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat     <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE, missing_data = "mcar"))</pre>
```

lsirm2pl\_mcar\_ss

2PL LSIRM with model selection approach for missing completely at random data.

## Description

lsirm2pl\_mar\_ss is used to fit 2PL LSIRM based on spike-and-slab priors in incomplete data assumed to be missing completely at random. lsirm2pl\_mar\_ss factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_mcar_ss(
   data,
   ndim = 2,
   niter = 15000,
   nburn = 2500,
   nthin = 5,
   nprint = 500,
   jump_beta = 0.4,
   jump_theta = 1,
   jump_alpha = 1,
```

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```
jump_gamma = 1,
  jump_z = 0.5,
 jump_w = 0.5,
 pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_spike_mean = -3,
 pr_spike_sd = 1,
 pr_slab_mean = 0.5,
 pr_slab_sd = 1,
 pr_mean_alpha = 0.5,
 pr_sd_alpha = 1,
 pr_a_theta = 0.001,
 pr_b_teta = 0.001,
 pr_xi_a = 1,
 pr_xi_b = 1,
 missing.val = 99,
 verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.

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pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable $xi$ . Default is $1$ .
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_mcar\_ss models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{i,i} = 1 | \theta_i, \alpha_i, \beta_i, \gamma, z_i, w_i)) = \theta_i * \alpha_i + \beta_i - \gamma ||z_i - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm2pl\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm2pl\_mar\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

90 lsirm2pl\_mcar\_ss

gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

## **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mcar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE, missing_data = "mcar"))</pre>
```

```
lsirm2pl_normal_fixed_gamma
```

2PL LSIRM fixing gamma to 1 with normal likelihood

## **Description**

lsirm2pl\_normal\_fixed\_gamma is used to fit 2PL LSIRM with gamma fixed to 1 for continuous variable. lsirm2pl\_normal\_fixed\_gamma factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_fixed_gamma(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
  pr_a_{eps} = 0.001,
 pr_b_{eps} = 0.001,
  verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.

nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is $0.001$ .
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is $0.001$ .
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## **Details**

lsirm2pl\_normal\_fixed\_gamma models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ 

### Value

lsirm2pl\_normal\_fixed\_gamma returns an object of list containing the following components:

data A data frame or matrix containing the variables used in the model.

bic A numeric value representing the Bayesian Information Criterion (BIC).

Details about the number of MCMC iterations, burn-in periods, and thinning mcmc\_inf intervals. The log maximum a posteriori (MAP) value and the iteration number at which map\_inf this MAP value occurs. beta\_estimate Posterior estimates of the beta parameter. theta\_estimate Posterior estimates of the theta parameter. sigma\_theta\_estimate Posterior estimates of the standard deviation of theta. z\_estimate Posterior estimates of the z parameter. w\_estimate Posterior estimates of the w parameter. beta Posterior samples of the beta parameter. theta Posterior samples of the theta parameter. theta\_sd Posterior samples of the standard deviation of theta. Posterior samples of the z parameter, represented as a 3-dimensional matrix z where the last axis denotes the dimension of the latent space. Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space. accept\_beta Acceptance ratio for the beta parameter. accept\_theta Acceptance ratio for the theta parameter. accept\_z Acceptance ratio for the z parameter. accept\_w Acceptance ratio for the w parameter. sigma\_estimate Posterior estimates of the standard deviation. sigma Posterior samples of the standard deviation.

### **Examples**

alpha

accept\_alpha

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

lsrm_result <- lsirm2pl_normal_fixed_gamma(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE))</pre>
```

Posterior estimates of the alpha parameter.

Acceptance ratio for the alpha parameter.

alpha\_estimate Posterior estimates of the alpha parameter.

```
lsirm2pl_normal_fixed_gamma_mar

2PL LSIRM fixing gamma to 1 with normal likelihood for missing at random data.
```

## Description

lsirm2pl\_normal\_fixed\_gamma\_mar is used to fit 2PL LSIRM with gamma fixed to 1 for continuous variable in incomplete data assumed to be missing at random.

lsirm2pl\_normal\_fixed\_gamma\_mar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_normal_fixed_gamma_mar(
  data,
  ndim = 2,
 niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
 pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
 pr_sd_alpha = 1,
  pr_a_theta = 0.001
  pr_b_t = 0.001,
  pr_a_{eps} = 0.001,
 pr_b_{eps} = 0.001,
 missing.val = 99,
  verbose = FALSE
)
```

#### **Arguments**

data

Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding

	item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is $0.001$ .
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood Default is $0.001$ .
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## **Details**

lsirm2pl\_normal\_fixed\_gamma\_mar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0,\sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

### Value

lsirm2pl\_normal\_fixed\_gamma\_mar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter.w\_estimate Posterior estimates of the w parameter.

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

sigma\_estimate Posterior estimates of the standard deviation.

sigma Posterior samples of the standard deviation.

alpha\_estimate Posterior estimates of the alpha parameter.

alpha Posterior estimates of the alpha parameter.

### **Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_fixed_gamma_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE, missing_data = "mar"))</pre>
```

lsirm2pl\_normal\_fixed\_gamma\_mcar

2PL LSIRM fixing gamma to 1 with normal likelihood for missing completely at random data.

### **Description**

lsirm2pl\_normal\_fixed\_gamma\_mcar is used to fit 2PL LSIRM with gamma fixed to 1 for continuous variable in incomplete data assumed to be missing completely at random.

Isirm2pl\_normal\_fixed\_gamma\_mcar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
```

```
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_b_eps = 0.001,
missing.val = 99,
verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.

pr\_b\_eps Numeric; the scale parameter of inverse gamma prior for variance of data likeli-

hood. Default is 0.001.

missing.val Numeric; a number to replace missing values. Default is 99.

verbose Logical; If TRUE, MCMC samples are printed for each nprint. Default is

FALSE.

### **Details**

lsirm2pl\_normal\_fixed\_gamma\_mcar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

### Value

lsirm2pl\_normal\_fixed\_gamma\_mcar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

z\_estimate Posterior estimates of the z parameter.
 w\_estimate Posterior estimates of the w parameter.
 beta Posterior samples of the beta parameter.
 theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

```
accept_theta Acceptance ratio for the theta parameter.

accept_z Acceptance ratio for the z parameter.

accept_w Acceptance ratio for the w parameter.

sigma_estimate Posterior estimates of the standard deviation.

sigma Posterior samples of the standard deviation.

alpha_estimate Posterior estimates of the alpha parameter.

alpha Posterior estimates of the alpha parameter.

Acceptance ratio for the alpha parameter.
```

### **Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_fixed_gamma_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE, missing_data = "mcar"))</pre>
```

lsirm2pl\_normal\_mar

2PL LSIRM with normal likelihood and missing at random data.

## **Description**

lsirm2pl\_normal\_mar is used to fit 2PL LSIRM for continuous variable in incomplete data assumed to be missing at random. lsirm2pl\_normal\_mar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_normal_mar(
  data,
  ndim = 2,
  niter = 15000,
```

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```
nburn = 2500,
 nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
 pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_mean_gamma = 0.5,
 pr_sd_gamma = 1,
 pr_mean_alpha = 0.5,
 pr_sd_alpha = 1,
 pr_a_theta = 0.001,
 pr_b_t = 0.001,
 pr_a_{eps} = 0.001,
 pr_b_{eps} = 0.001,
 missing.val = 99,
 verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.

pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_normal\_mar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{i,i} = \theta_i + \beta_i - \gamma ||z_i - w_i|| + e_{i,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

### Value

lsirm2pl\_normal\_mar returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

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gaiiiiia_estiiiate	posterior estilliates of gainina parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
	TO 1 1111 OI

ma actimate masteriar astimates of samma parameter

imp\_estimate Probability of imputating a missing value with 1.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.
gamma Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

imp Imputation for missing Values using posterior samples.

Acceptance ratio for the beta parameter. accept\_beta accept\_theta Acceptance ratio for the theta parameter. accept\_z Acceptance ratio for the z parameter. accept\_w Acceptance ratio for the w parameter. accept\_gamma Acceptance ratio for the gamma parameter. sigma\_estimate Posterior estimates of the standard deviation. Posterior samples of the standard deviation. sigma alpha\_estimate Posterior estimates of the alpha parameter. alpha Posterior estimates of the alpha parameter. Acceptance ratio for the alpha parameter. accept\_alpha

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

### **Examples**

```
lsirm2pl_normal_mar_ss
```

2pl LSIRM with normal likelihood and model selection approach for missing at random data.

### **Description**

Isirm2pl\_normal\_mar\_ss is used to fit 2pl LSIRM with model selection approach based on spike-and-slab priors for continuous variable in incomplete data assumed to be missing at random. Isirm2pl\_normal\_mar\_ss factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1pl model, 2pl model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_normal_mar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_spike_mean = -3,
  pr_spike_sd = 1,
  pr_slab_mean = 0.5,
  pr_slab_sd = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_{eps} = 0.001,
  pr_b_{eps} = 0.001,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
  pr_xi_a = 0.001,
  pr_xi_b = 0.001,
 missing.val = 99,
```

```
verbose = FALSE
)
```

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood Default is 0.001.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.

pr\_xi\_b Numeric; the second shape parameter of beta prior for latent variable xi. Default

is 1.

missing.val Numeric; a number to replace missing values. Default is 99.

verbose Logical; If TRUE, MCMC samples are printed for each nprint. Default is

FALSE.

### **Details**

Isirm2pl\_normal\_mar\_ss models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0,\sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm2pl\_normal\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

#### Value

lsirm2pl\_normal\_mar\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

gamma Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = $0$ , otherwise latent space model with gamma > $0$ .
<pre>imp_estimate</pre>	Probability of imputating a missing value with 1.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

## **Examples**

```
# generate example (continuous) item response matrix
data     <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat      <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_mar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE, missing_data = "mar"))</pre>
```

lsirm2pl\_normal\_mcar

2PL LSIRM with normal likelihood and missing completely at random data.

### Description

lsirm2pl\_normal\_mcar is used to fit 2PL LSIRM for continuous variable in incomplete data assumed to be missing completely at random. lsirm2pl\_normal\_mcar factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_normal_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
 pr_a_{eps} = 0.001,
 pr_b_eps = 0.001,
 missing.val = 99,
  verbose = FALSE
)
```

### **Arguments**

data

Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding

	item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_normal\_mcar models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

#### Value

lsirm2pl\_normal\_mcar returns an object of list containing the following components:

data A data frame or matrix containing the variables used in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

Acceptance ratio for the beta parameter. accept\_beta accept\_theta Acceptance ratio for the theta parameter. Acceptance ratio for the z parameter. accept\_z accept\_w Acceptance ratio for the w parameter. Acceptance ratio for the gamma parameter. accept\_gamma sigma\_estimate Posterior estimates of the standard deviation. sigma Posterior samples of the standard deviation. alpha\_estimate Posterior estimates of the alpha parameter. alpha Posterior estimates of the alpha parameter. accept\_alpha Acceptance ratio for the alpha parameter.

#### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

### **Examples**

lsirm2pl\_normal\_mcar\_ss

2PL LSIRM with normal likelihood and model selection approach for missing completely at random data.

### **Description**

lsirm2pl\_normal\_mcar\_ss is used to fit 2PL LSIRM with model selection approach based on spike-and-slab priors for continuous variable in incomplete data assumed to be missing completely at random. lsirm2pl\_normal\_mcar\_ss factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_mcar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
```

```
jump\_theta = 1,
  jump_alpha = 1,
  jump\_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
 pr_mean_beta = 0,
 pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_spike_mean = -3,
 pr_spike_sd = 1,
 pr_slab_mean = 0.5,
 pr_slab_sd = 1,
 pr_mean_alpha = 0.5,
 pr_sd_alpha = 1,
 pr_a_{eps} = 0.001,
 pr_b_{eps} = 0.001,
 pr_a_theta = 0.001,
 pr_b_t = 0.001,
 pr_xi_a = 0.001,
 pr_xi_b = 0.001,
 missing.val = 99,
 verbose = FALSE
)
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.

pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is $0.001$ .
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is $0.001$ .
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable $xi$ . Default is $1$ .
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_normal\_mcar\_ss models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0,\sigma^2)$  Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References. lsirm2pl\_normal\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

#### Value

lsirm2pl\_normal\_mcar\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

missing.val A number to replace missing values.

bic Numeric value with the corresponding BIC.

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.

theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.
theta Posterior samples of the theta parameter.

gamma Posterior samples of the gamma parameter.

theta\_sd Posterior samples of the standard deviation of theta.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

pi Posterior samples of phi which is indicator of spike and slab prior. If phi is 1,

log gamma follows the slab prior, otherwise follows the spike prior.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

sigma\_estimate Posterior estimates of the standard deviation.

sigma Posterior samples of the standard deviation.

alpha\_estimate Posterior estimates of the alpha parameter.

alpha Posterior estimates of the alpha parameter.

### References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

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### **Examples**

lsirm2pl\_normal\_o

2PL LSIRM with normal likelihood

# Description

lsirm2pl\_normal\_o is used to fit 2PL LSIRM for continuous variable. lsirm2pl\_normal\_o factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_o(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
```

```
pr_sd_gamma = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
verbose = FALSE
)
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is $0.001.$

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pr\_b\_eps Numeric; the scale parameter of inverse gamma prior for variance of data likeli-

hood. Default is 0.001.

verbose Logical; If TRUE, MCMC samples are printed for each nprint. Default is

FALSE.

#### **Details**

Isirm2pl\_normal\_o models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_i$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ 

#### Value

lsirm2pl\_normal\_o returns an object of list containing the following components:

data Data frame or matrix containing the variables used in the model.

bic A numeric value representing the Bayesian Information Criterion (BIC).

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

```
accept_w Acceptance ratio for the w parameter.
accept_gamma Acceptance ratio for the gamma parameter.
sigma_estimate Posterior estimates of the standard deviation.
alpha_estimate Posterior estimates of the alpha parameter.
alpha Posterior estimates of the alpha parameter.
accept_alpha Acceptance ratio for the alpha parameter.
```

### **Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)
lsirm_result <- lsirm2pl_normal_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE))</pre>
```

lsirm2pl\_normal\_ss

2PL LSIRM with normal likelihood and model selection approach.

### Description

lsirm2pl\_normal\_ss is used to fit 2PL LSIRM for continuous variable with model selection approach. lsirm2pl\_normal\_ss factorizes item response matrix into column-wise item effect, rowwise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_ss(
    data,
    ndim = 2,
    niter = 15000,
    nburn = 2500,
    nthin = 5,
    nprint = 500,
    jump_beta = 0.4,
    jump_theta = 1,
    jump_alpha = 1,
    jump_gamma = 1,
    jump_z = 0.5,
    jump_w = 0.5,
    pr_mean_beta = 0,
```

```
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 0.001,
pr_xi_b = 0.001,
verbose = FALSE
```

# Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; mean of spike prior for log gamma default value is -3.
pr_spike_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_slab_mean	Numeric; mean of spike prior for log gamma default value is 0.5.
pr_slab_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.

pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_eps	Numeric; shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

#### **Details**

lsirm2pl\_normal\_ss models the continuous value of response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . lsrm2pl\_noraml\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

#### Value

lsirm2pl\_normal\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables in the model.

bic Numeric value with the corresponding BIC.

mcmc\_inf number of mcmc iteration, burn-in periods, and thinning intervals.

map\_inf value of log maximum a posterior and iteration number which have log maxi-

mum a posterior.

beta\_estimate posterior estimation of beta. theta\_estimate posterior estimation of theta.

 $sigma\_theta\_estimate$ 

posterior estimation of standard deviation of theta.

sigma\_estimate posterior estimation of standard deviation.

gamma\_estimate posterior estimation of gamma.

z\_estimate posterior estimation of z.w\_estimate posterior estimation of w.

posterior estimation of phi. inclusion probability of gamma. if estimation of phi pi\_estimate

is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space

model with gamma > 0.

posterior samples of beta. beta theta posterior samples of theta.

theta\_sd posterior samples of standard deviation of theta.

sigma posterior samples of standard deviation.

posterior samples of gamma. gamma

posterior samples of z. The output is 3-dimensional matrix with last axis repre-Z

sent the dimension of latent space.

posterior samples of w. The output is 3-dimensional matrix with last axis repre-

sent the dimension of latent space.

posterior samples of phi which is indicator of spike and slab prior. If phi is 1, рi

log gamma follows the slab prior, otherwise follows the spike prior.

accept\_beta accept ratio of beta. accept\_theta accept ratio of theta. accept\_w accept ratio of w. accept\_z accept ratio of z.

accept\_gamma accept ratio of gamma.

alpha\_estimate Posterior estimates of the alpha parameter.

alpha Posterior estimates of the alpha parameter. Acceptance ratio for the alpha parameter.

#### References

accept\_alpha

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

### **Examples**

```
# generate example (continuous) item response matrix
         <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)
lsirm_result <- lsirm2pl_normal_ss(data)</pre>
# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE))</pre>
```

lsirm2pl\_o

lsirm2pl\_o

2PL LSIRM.

### **Description**

lsirm2pl\_o is used to fit 2PL LSIRM. lsirm2pl\_o factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

# Usage

```
lsirm2pl_o(
  data,
  ndim = 2,
 niter = 15000,
 nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump\_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
 pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
 pr_a_theta = 0.001,
 pr_b_ten = 0.001,
  verbose = FALSE
)
```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.

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nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is $0.001$ .
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_o models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

### Value

lsirm2pl\_o returns an object of list containing the following components:

Data frame or matrix containing the variables used in the model.

bic A numeric value representing the Bayesian Information Criterion (BIC).

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning intervals.

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map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.

theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.
 w\_estimate Posterior estimates of the w parameter.
 beta Posterior samples of the beta parameter.
 theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.
accept\_theta Acceptance ratio for the theta parameter.
accept\_z Acceptance ratio for the z parameter.

Acceptance ratio for the w parameter.

 ${\tt alpha\_estimate} \ \ Posterior\ estimates\ of\ the\ alpha\ parameter.$ 

alpha Posterior estimates of the alpha parameter.

### **Examples**

accept\_w

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

lsirm_result <- lsirm2pl_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE))</pre>
```

lsirm2pl\_ss 125

lsirm2pl\_ss

2PL LSIRM with model selection approach.

#### **Description**

lsirm2pl\_ss is used to fit 2PL LSIRM with model selection approach based on spike-and-slab priors. lsirm2pl\_ss factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump\_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
 pr_mean_theta = 0,
 pr_spike_mean = -3,
 pr_spike_sd = 1,
  pr_slab_mean = 0.5,
  pr_slab_sd = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_t = 0.001,
 pr_xi_a = 1,
 pr_xi_b = 1,
  verbose = FALSE
)
```

### **Arguments**

data

Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.

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ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### **Details**

lsirm2pl\_ss models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item i and latent position  $z_j$  of respondent j in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

lsirm2pl\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

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#### Value

lsirm2pl\_ss returns an object of list containing the following components:

data Data frame or matrix containing the variables used in the model.

bic A numeric value representing the Bayesian Information Criterion (BIC).

mcmc\_inf Details about the number of MCMC iterations, burn-in periods, and thinning

intervals.

map\_inf The log maximum a posteriori (MAP) value and the iteration number at which

this MAP value occurs.

beta\_estimate Posterior estimates of the beta parameter.
theta\_estimate Posterior estimates of the theta parameter.

sigma\_theta\_estimate

Posterior estimates of the standard deviation of theta.

gamma\_estimate Posterior estimates of gamma parameter.

z\_estimate Posterior estimates of the z parameter.

w\_estimate Posterior estimates of the w parameter.

beta Posterior samples of the beta parameter.

theta Posterior samples of the theta parameter.

theta\_sd Posterior samples of the standard deviation of theta.

gamma Posterior samples of the gamma parameter.

z Posterior samples of the z parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

w Posterior samples of the w parameter, represented as a 3-dimensional matrix

where the last axis denotes the dimension of the latent space.

accept\_beta Acceptance ratio for the beta parameter.

accept\_theta Acceptance ratio for the theta parameter.

accept\_z Acceptance ratio for the z parameter.

accept\_w Acceptance ratio for the w parameter.

accept\_gamma Acceptance ratio for the gamma parameter.

pi\_estimate Posterior estimation of phi. inclusion probability of gamma. if estimation of phi

is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space

model with gamma > 0.

pi Posterior samples of phi which is indicator of spike and slab prior. If phi is 1,

log gamma follows the slab prior, otherwise follows the spike prior.

alpha\_estimate Posterior estimates of the alpha parameter.

alpha Posterior estimates of the alpha parameter.

accept\_alpha Acceptance ratio for the alpha parameter.

#### References

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. The Annals of Statistics, 33(2), 730-773.

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### **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

lsirm_result <- lsirm2pl_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE))</pre>
```

onepl

1PL Rasch model.

# Description

onepl is used to fit 1PL Rasch model.

# Usage

```
onepl(
   data,
   niter = 15000,
   nburn = 2500,
   nthin = 5,
   nprint = 500,
   jump_beta = 0.4,
   jump_theta = 1,
   pr_mean_beta = 0,
   pr_sd_beta = 1,
   pr_mean_theta = 0,
   pr_a_theta = 0.001,
   pr_b_theta = 0.001
```

### **Arguments**

data	Matrix; binary item response matrix to be analyzed. Each row is assumed to be respondent and its column values are assumed to be response to the corresponding item.
niter	Numeric; number of iterations to run MCMC sampling. default value is 15000.
nburn	Numeric; number of initial, pre-thinning, MCMC iterations to discard. default value is 2500.
nthin	Numeric;number of thinning, MCMC iterations to discard. default value is 5.
nprint	Numeric; MCMC samples is displayed during execution of MCMC chain for each nprint. default value is 500.
jump_beta	Numeric; jumping rule of the proposal density for beta. default value is 0.4.

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jump_theta	Numeric; jumping rule of the proposal density for theta. default value is 1.0.
pr_mean_beta	Numeric; mean of normal prior for beta. default value is 0.
pr_sd_beta	Numeric; standard deviation of normal prior for beta. default value is 1.0.
pr_mean_theta	Numeric; mean of normal prior for theta. default value is 0.
pr_a_theta	Numeric; shape parameter of inverse gamma prior for variance of theta. default value is 0.001.
pr_b_theta	Numeric; scale parameter of inverse gamma prior for variance of theta. default value is 0.001.

### **Details**

onep1 models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i)) = \theta_j + \beta_i$$

### Value

onepl returns an object of list containing the following components:

```
beta_estimate posterior estimation of beta.

theta_estimate posterior estimation of theta.

sigma_theta_estimate posterior estimation of standard deviation of theta.

beta posterior samples of beta.

theta posterior samples of theta.

theta_sd posterior samples of standard deviation of theta.

accept_beta accept ratio of beta.

accept_theta accept ratio of theta.
```

# Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
result <- onepl(data)</pre>
```

plot

plot	Plotting the interaction map or summarizing the parameter estimate
	of fitted LSIRM with box plot.

# Description

plot is used to plot the interaction map of fitted LSIRM or summarizing the parameter estimate of fitted LSIRM with box plot.

# Usage

```
plot(
  object,
  ...,
  option = "interaction",
  rotation = FALSE,
  cluster = NA,
  which.clust = "item",
  interact = FALSE,
  chain.idx = 1
)
```

# Arguments

object	Object of class 1sirm.
	Additional arguments for the corresponding function.
option	Character; If value is "interaction", draw the interaction map that represents interactions between respondents and items. If value is "beta", draw the boxplot for the posterior samples of beta. If value is "theta", draw the distribution of the theta estimates per total test score for the data. If value is "alpha", draw the boxplot for the posterior samples of alpha. The "alpha" is only available for 2PL LSIRM.
rotation	Logical; If TRUE the latent positions are visualized after oblique (oblimin) rotation.
cluster	Character; If value is "neyman" the cluster result are visualized by Point Process Cluster Analysis. If value is "spectral", spectral clustering method applied. Default is NA.
which.clust	Character; Choose which values to clustering. "resp" is the option for respondent and "item" is the option for items. Default is "item".
interact	Logical; If TRUE, draw the interaction map interactively.
chain.idx	Numeric; Index of MCMC chain. Default is 1.

# Value

plot returns the interaction map or boxplot for parameter estimate.

print.summary.lsirm 131

### **Examples**

```
# generate example item response matrix
data     <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)
lsirm_result <- lsirm(data ~ lsirm1pl())
plot(lsirm_result)

# use oblique rotation
plot(lsirm_result, rotation = TRUE)

# interaction map interactively
plot(lsirm_result, interact = TRUE)

# clustering the respondents or items
plot(lsirm_result, cluster = TRUE)</pre>
```

print.summary.lsirm

Print the summary the result of LSIRM

### **Description**

print.summary.lsirm is used to print summary the result of LSIRM.

# Usage

```
## S3 method for class 'summary.lsirm'
print(x, ...)
```

### **Arguments**

- x List; summary of LSIRM with summary.lsirm.
- ... Additional arguments.

### Value

```
print.summary.lsirm return a summary of LSIRM.
```

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
lsirm_result <- lsirm(data ~ lsirm1pl())
summary(lsirm_result)</pre>
```

summary.lsirm

summary.lsirm	Summary the result of LSIRM	
---------------	-----------------------------	--

### **Description**

summary is used to summary the result of LSIRM.

# Usage

```
## S3 method for class 'lsirm'
summary(object, chain.idx = 1, estimate = "mean", CI = 0.95, ...)
```

# Arguments

object	Object of class lsirm.
chain.idx	Numeric; Index of MCMC chain. Default is 1.
estimate	Character; Specifies the type of posterior estimate to provide for beta parameters. Options are "mean", "median", or "mode". Default is "mean".
CI	Numeric; The significance level for the highest posterior density interval (HPD) for the beta parameters. Default is 0.95.
	Additional arguments.

### Value

summary.lsirm contains following elements. A print method is available.

call	R call used to fit the model.
coef	Covariate coefficients posterior means.
mcmc.opt	The number of mcmc iteration, burn-in periods, and thinning intervals.
map.inf	Value of log maximum a posterior and iteration number which have log maximum a posterior.
BIC	Numeric value with the corresponding Bayesian information criterion (BIC).
method	Which model is fitted.
missing	The assumed missing type. One of NA, "mar" and "mcar".
dtype	Type of input data (Binary or Continuous).
SS	Whether a model selection approach using the spike-slab prior is applied.

# Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
# 1PL LSIRM object
lsirm_result <- lsirm(data ~ lsirm1pl())
summary(lsirm_result)</pre>
```

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**TDRI** 

Inductive Reasoning Developmental Test

### **Description**

TDRI dataset is the answer to Inductive Reasoning Developmental Test of 1,803 Brazilians with age varying from 5 to 85 years.

### Usage

```
data(TDRI)
```

#### **Format**

A binary matrix with 1,803 rows and 56 columns.

### **Details**

It presents data from 1,803 Brazilians (52.5% female) with age varying from 5 to 85 years (M = 15.75; SD = 12.21) that answered to the Inductive Reasoning Developmental Test – IRDT, with 56 items designed to assess developmentally sequenced and hierarchically organized inductive reasoning.

### **Source**

https://figshare.com/articles/dataset/TDRI\_dataset\_csv/3142321

twopl

2PL Rasch model.

# Description

twopl is used to fit 2PL Rasch model. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect.

### Usage

```
twopl(
  data,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
```

twopl

```
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001)
```

# Arguments

data	Matrix; binary item response matrix to be analyzed. Each row is assumed to be respondent and its column values are assumed to be response to the corresponding item.
niter	Numeric; number of iterations to run MCMC sampling. default value is 15000.
nburn	Numeric; number of initial, pre-thinning, MCMC iterations to discard. default value is 2500.
nthin	Numeric;number of thinning, MCMC iterations to discard. default value is 5.
nprint	Numeric; MCMC samples is displayed during execution of MCMC chain for each nprint. default value is 500.
jump_beta	Numeric; jumping rule of the proposal density for beta. default value is 0.4.
jump_theta	Numeric; jumping rule of the proposal density for theta. default value is 1.0.
jump_alpha	Numeric; jumping rule of the proposal density for alpha default value is 1.0.
pr_mean_beta	Numeric; mean of normal prior for beta. default value is 0.
pr_sd_beta	Numeric; standard deviation of normal prior for beta. default value is 1.0.
pr_mean_theta	Numeric; mean of normal prior for theta. default value is 0.
pr_mean_alpha	Numeric; mean of normal prior for alpha. default value is 0.5.
pr_sd_alpha	Numeric; mean of normal prior for beta. default value is 1.0.
pr_a_theta	Numeric; shape parameter of inverse gamma prior for variance of theta. default value is $0.001$ .
pr_b_theta	Numeric; scale parameter of inverse gamma prior for variance of theta. default value is 0.001.

### **Details**

twopl models the probability of correct response by respondent j to item i with item effect  $\beta_i$ , respondent effect  $\theta_j$ . For 2pl model, the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$logit(P(Y_{j,i} = 1 | \theta_j, \beta_i, \alpha_i)) = \theta_j * \alpha_i + \beta_i$$

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### Value

twop1 returns an object of list containing the following components:

beta\_estimate posterior estimation of beta. theta\_estimate posterior estimation of theta. sigma\_theta\_estimate

posterior estimation of standard deviation of theta.

alpha\_estimate posterior estimation of alpha.
beta posterior samples of beta.
theta posterior samples of theta.

theta\_sd posterior samples of standard deviation of theta.

alpha posterior samples of alpha.

accept\_beta accept ratio of beta.
accept\_theta accept ratio of theta.
accept\_alpha accept ratio of alpha.

# **Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)
result <- twopl(data)</pre>
```

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