

intcensROC

Fast Spline Based Sieve AUC Estimator for Interval Censored Data

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This document provides an comprehensive example for using the `intcensROC` package to estimate the receiver operating characteristic (ROC) curve and time-dependent area under the curve (AUC) for interval censored survival data, that is not not applicable for existing methods.

The estimator applies a generalized gradient projection method on Spline based likelihood function to obtain the joint distribution function between survival time and biomarker and compute the ROC curve and time-dependent AUC with the estimated joint distribution function. Features of this package include:

- 1 The algorithm is implemented in C++, and ported to R by Rcpp, to facilitate fast computation.
- 2 The estimator uses a constrained minimization method and is designed for the interval survival data.

Function Signature and Return of `intcensROC`

Function to compute ROC curve

```
res <- intcensROC(U, V, Marker, Delta, PredictTime, gridNumber = 500)
```

Function arguments:

U: An array contains left times of the censored intervals for the sample.

V: An array contains right times of the censored intervals for the sample.

Marker: An array contains marker levels for the samples.

Delta: An array of indicator for the censoring type, use 1, 2, 3 for left, interval and right censoring types, correspondingly.

PredictTime: A scalar for predict time for the ROC.

gridNumber: A integer for the number of grid of the ROC curve, the default value is 500.

Function return:

A dataframe contains two columns

tp: A array for true positive rate.

fp: A array for false positive rate.

Function Signature and Return of `intcensAUC`

Function to compute AUC

```
auc <- intcensAUC(ROCdata)
```

Function argument:

`ROCdata`: A dataframe from the function `intcensROC`.

Function return:

`auc`: A scalar for AUC.

A Simple Start Off Example

A start off example to use function `intcensROC` and `intcensAUC`

```
library(intcensROC)
## example interval censored data
U <- runif(100, min = 0.1, max = 5)
V <- runif(100, min = 0.1, max = 5) + U
Marker <- runif(100, min = 5, max = 10)
Delta <- sample.int(3, size = 100, replace = TRUE)
pTime <- 4
## compute the ROC curve
res <- intcensROC(U, V, Marker, Delta, pTime, gridNumber = 500)
head(res)

##           fp           tp
## 1 2.655677e-07 7.096877e-07
## 2 5.762095e-07 1.481246e-06
## 3 9.772144e-07 2.377129e-06
## 4 1.513871e-06 3.459788e-06
## 5 2.231469e-06 4.791677e-06
## 6 3.175296e-06 6.435248e-06

##compute the AUC
auc <- intcensAUC(res)
print(auc)

## [1] 0.5984202
```

Example-Background

Here, we present a comprehensive example as a tutorial on how to use `intcensROC` package.

- We assume the survival time T follows an exponential distribution with hazard rate $\lambda = \frac{\log(2)}{24}$.
- The marker M is assumed to follow a beta distribution with parameter $\alpha = 2.35$ and $\beta = 1.87$.
- The joint distribution of (T, M) is assumed to be generated by Clayton Coupla with parameter $\alpha > 1$

$$F_{T,M}(t, m) = Pr(T < t, M < m) = \{F_T(t)^{\alpha-1} + F_M(m)^{\alpha-1} - 1\}^{\frac{1}{\alpha-1}}$$

Here $F_T(\cdot)$ and $F_M(\cdot)$ denote the distribution functions of T and M respectively. The dependence between T and M is denote by Kendall $\tau = \frac{\alpha-1}{\alpha+1}$

- The random assessment interval $[U, V]$ are sampled from uniform distribution, V 's are sampled within $[L_0, L_c]$, and U is generated from uniform distribution on $[0, V - L_0]$. Here L_c is determined by the censoring rate $\rho = 0.3$ and $L_0 = 0.1$ is the minimum time difference between U and V .

Data Simulation

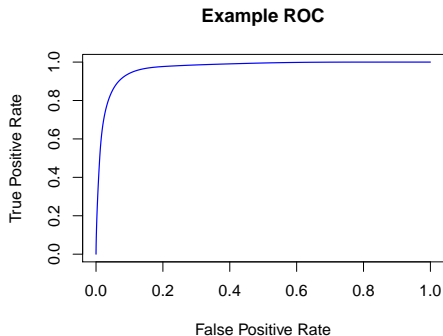
```
library("copula")
f <- function(x, L0, rate, censor){
  1/((x-L0)*rate)*exp(-L0*rate)-1/((x-L0)*rate)*exp(-x*rate)-censor
}
dataSim <- function(kendall_tau = 0.3, n = 100, rho = 0.3, lambda = log(2)/6){
  b_alpha <- 2.35
  b_beta <- 1.87
  scale <- 10
  kendall_tau <- iTau( claytonCopula(), kendall_tau)
  Int_cop <- claytonCopula(param = kendall_tau, dim = 2)
  Int_mvdc <- mvdc(Int_cop, c("exp","beta"), paramMargins =
    list(list(rate = lambda), list(shape1=b_alpha,shape2=b_beta)))
  Int_obs_data <- rMvdc(n, Int_mvdc)
  colnames(Int_obs_data) <- c("event_time", "marker")
  Int_obs_data[, "marker"] <- Int_obs_data[, "marker"]*scale
  L0 <- 0.1
  size <- n
  U <- rep(0, size)
  L <- uniroot(f, lower = 10^(-6), upper = 500, tol=0.000001,
    L0=L0, rate=lambda, censor=rho)
  V <- runif(size, L0, L$root)
  for (i in 1:size) U[i] <- runif(1, 0, (V[i]-L0))
  delta_1 <- Int_obs_data[, "event_time"] < U
  delta_2 <- Int_obs_data[, "event_time"] >= U &&
  Int_obs_data[, "event_time"] <= V
  delta_3 <- Int_obs_data[, "event_time"] > V
  data <- data.frame(U = U, V = V, delta = delta_1+2*delta_2+3*delta_3,
    marker=Int_obs_data[, "marker"])
}
```


Compute ROC and AUC

```
mydata <- dataSim(kendall_tau = 0.7, n = 300, rho = 0.3, lambda = log(2)/24)
roc     <- intcensROC(U=mydata[,"U"],V=mydata[,"V"], Marker=mydata[,"marker"],
                    Delta=mydata[,"delta"], PredictTime=12)
print(intcensAUC(roc))
```

```
## [1] 0.9697949
```

```
plot(roc$fp, roc$tp, type = "l", lwd = 1.2, col="blue", main = "Example ROC",
     xlab = "False Positive Rate", ylab = "True Positive Rate" )
```



Session Information

- R version 4.0.2 (2020-06-22), x86_64-pc-linux-gnu
- Running under: Ubuntu 18.04.5 LTS
- Matrix products: default
- BLAS: /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.7.1
- LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.7.1
- Base packages: base, datasets, grDevices, graphics, methods, stats, utils
- Other packages: copula 1.0-1, intcensROC 0.1.3, knitr 1.33
- Loaded via a namespace (and not attached): ADGofTest 0.3, Matrix 1.2-18, Rcpp 1.0.5, compiler 4.0.2, evaluate 0.14, grid 4.0.2, gsl 2.1-6, highr 0.8, lattice 0.20-41, magrittr 1.5, mvtnorm 1.1-1, numDeriv 2016.8-1.1, pcaPP 1.9-74, pracma 2.3.3, pspline 1.0-18, stabledist 0.7-1, stats4 4.0.2, stringi 1.5.3, stringr 1.4.0, tools 4.0.2, xfun 0.23

```
## [1] "Start Time Fri Jun 11 09:47:18 2021"  
## [1] "End Time Fri Jun 11 09:47:19 2021"
```