

# Package ‘equateMultiple’

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**Type** Package

**Title** Equating of Multiple Forms

**Version** 1.1.1

**Author** Michela Battauz [aut, cre],  
Waldir Leoncio [ctb]

**Maintainer** Michela Battauz <michela.battauz@uniud.it>

**Description** Equating of multiple forms using Item Response Theory (IRT) methods (Battauz M. (2017) <[doi:10.1007/s11336-016-9517-x](https://doi.org/10.1007/s11336-016-9517-x)>, Battauz and 'Leoncio' (2023) <[doi:10.1177/01466216231151702](https://doi.org/10.1177/01466216231151702)>, Haberman S. J. (2009) <[doi:10.1002/j.2333-8504.2009.tb02197.x](https://doi.org/10.1002/j.2333-8504.2009.tb02197.x)>).

**License** GPL-3

**Imports** stats, graphics, numDeriv, statmod, Rcpp (>= 1.0.7), Matrix,  
mvtnorm, data.table

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equateMultiple-package

*Equating of Multiple Forms*

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## Description

The **EquateMultiple** package implements IRT-based methods to equate simultaneously many forms calibrated separately. This package estimates the equating coefficients to convert the item parameters and the ability values to the scale of the base form. It can be applied to a large number of test forms, as well as to 2 forms. The computation of the equated scores is also implemented.

## Details

This package implements the methods proposed in Haberman (2009), Battauz (2017) and Battauz and Leoncio (2023). Function `multiec` computes the equating coefficients to convert the item parameters and the ability values to the scale of the base form. The methods implemented are: multiple mean-geometric mean (Haberman, 2009), multiple mean-mean, multiple item response function, multiple test response function (Battauz, 2017), and likelihood-based linking (Battauz and Leoncio, 2023). The function provides the equating coefficients, estimates of item parameters on a common scale and the standard errors of the equating coefficients and item parameters on a common scale. Equated scores can be computed using true score equating and observed score equating methods. Standard errors of equated scores are also provided.

## Author(s)

Michela Battauz [aut, cre], Waldir Leoncio [ctb]

Maintainer: Michela Battauz <michela.battauz@uniud.it>

## References

Battauz, M. (2017). Multiple equating of separate IRT calibrations. *Psychometrika*, **82**, 610–636. doi:10.1007/s11336-016-9517-x.

Battauz, M., Leoncio, W. (2023). A Likelihood Approach to Item Response Theory Equating of Multiple Forms *Applied Psychological Measurement*, **47**, 200-220. doi: 10.1177/01466216231151702.

Haberman, S. J. (2009). Linking parameter estimates derived from an item response model through separate calibrations. ETS Research Report Series, 2009, i-9. doi:10.1002/j.2333-8504.2009.tb02197.x.

## See Also

[equateIRT](#)

**Examples**

```

data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)
# Estimation of the equating coefficients with the multiple mean-mean method
eqMM <- multiec(mods = mods, base = 1, method = "mean-mean")
summary(eqMM)

# Estimation of the equating coefficients with the
# multiple mean-geometric mean method (Haberman, 2009)
eqMGM <- multiec(mods = mods, base = 1, method = "mean-gmean")
summary(eqMGM)

# Estimation of the equating coefficients with the multiple item response function method
eqIRF <- multiec(mods = mods, base = 1, method = "irf")
summary(eqIRF)

# Estimation of the equating coefficients with the multiple item response function method
# using as initial values the estimates obtained with the multiple mean-geometric mean method
eqMGM <- multiec(mods = mods, base = 1, method = "mean-gmean", se = FALSE)
eqIRF <- multiec(mods = mods, base = 1, method = "irf", start = eqMGM)
summary(eqIRF)

# Estimation of the equating coefficients with the multiple test response function method
eqTRF <- multiec(mods = mods, base = 1, method = "trf")
summary(eqTRF)

# Estimation of the equating coefficients with the likelihood-based method
eqLIK <- multiec(mods = mods, base = 1, method = "lik")
summary(eqLIK)

# scoring using the true score equating method and equating coefficients
# obtained with the multiple item response function method
score(eqIRF)

```

---

eqc.mlteqc

*Extract Equating Coefficients of Multiple Forms*


---

**Description**

eqc is a generic function which extracts the equating coefficients.

**Usage**

```

## S3 method for class 'mlteqc'
eqc(x, ...)

```

**Arguments**

x                    object of the class mlteqc returned by function [multiec](#)  
...                   further arguments passed to or from other methods.

**Value**

A data frame containing the equating coefficients.

**Author(s)**

Michela Battauz

**See Also**

[multiec](#)

**Examples**

```
data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)
# Estimation of the equating coefficients with the multiple item response function method
eqIRF <- multiec(mods = mods, base = 1, method = "irf")

# extract equating coefficients
eqc(eqIRF)
```

---

item.common

*Estimates the Item Parameters on a Common Scale*

---

**Description**

item.common is a generic function which extracts the estimates of the item parameters on the scale of the base form, which is obtained using the item parameter estimates across all forms.

**Usage**

```
item.common(x, ...)
```

```
## S3 method for class 'mlteqc'
item.common(x, ...)
```

**Arguments**

x                    object of the class mlteqc returned by function [multiec](#)  
...                   further arguments passed to or from other methods.

**Value**

A data frame containing item names (`Item`), estimated value of the item parameter (`Estimate`) and its standard error (`StdErr`).

**Author(s)**

Michela Battauz

**See Also**

[itm](#), [multiec](#)

**Examples**

```
data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)
# Estimation of the equating coefficients with the multiple item response function method
eqIRF <- multiec(mods = mods, base = 1, method = "irf")

# estimates of the item parameters on the scale of the base form
item.common(eqIRF)
```

---

itm.mlteqc

*Extract Item Parameters*

---

**Description**

`itm` is a generic function which extracts a data frame containing the item parameters of multiple forms in the original scale and the item parameters converted to the scale of the base form.

**Usage**

```
## S3 method for class 'mlteqc'
itm(x, ...)
```

**Arguments**

`x` object of the class `mlteqc` returned by function [multiec](#)  
`...` further arguments passed to or from other methods.

**Value**

A data frame containing item names (`Item`), item parameters of all the forms (e.g. `T1`, ..., `T3`), and item parameters of all the forms converted in the scale of the base form (e.g. `T3.as.T1`).

**Author(s)**

Michela Battauz

**See Also**[multiec](#)**Examples**

```

data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)
# Estimation of the equating coefficients with the multiple item response function method
eqIRF <- multiec(mods = mods, base = 1, method = "irf")

# extract item parameters
itm(eqIRF)

```

---

mathTest	<i>Math Test Data</i>
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**Description**

List of 5 datasets with the binary responses (correct/wrong) to 5 forms of a math test.

**Usage**

```
data("mathTest")
```

**Format**

A list of length 5, containing 5 data frames. Each dataset contains rows of responses from individuals to various items, with the item labels as the column headers.

**Examples**

```

data(mathTest)
mathTest[[1]][1:3,]

```

---

multiec	<i>Multiple Equating Coefficients</i>
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---

**Description**

Calculates the equating coefficients between multiple forms.

**Usage**

```

multiec(mods, base = 1, method = "mean-mean", se = TRUE, nq = 30, start = NULL,
iter.max = 100000, obsinf = TRUE, trace = TRUE)

```

**Arguments**

mods	an object of the class <code>modIRT</code> containing item parameter coefficients and their covariance matrix of the forms to be equated.
base	integer value indicating the base form.
method	the method used to compute the equating coefficients. This should be one of "mean-mean", "mean-gmean", "irf", "trf" or "lik" (see details).
se	logical; if TRUE the standard errors of the equating coefficients and the item parameters on a common scale are computed.
nq	number of quadrature points used for the Gauss-Hermite quadrature for methods "irf" or "trf".
start	initial values. This can be a vector containing the A and B equating coefficients excluding the base form, or an object of class <code>mlteqc</code> returned by function <code>multiec</code> . Used only with methods "irf", "trf" and "lik".
iter.max	maximum number of iterations allowed. Used only with methods "irf", "trf" and "lik".
obsinf	logical; if TRUE the standard errors are computed using the observed information matrix. Used only with method "lik", see Battauz and Leoncio (2023).
trace	logical; if TRUE tracing information is produced.

**Details**

The methods implemented for the computation of the multiple equating coefficients are the multiple mean-mean method ("mean-mean"), the multiple mean-geometric mean method ("mean-gmean"), the multiple item response function method ("irf"), the multiple test response function method ("trf"), and likelihood-based equating ("lik").

**Value**

An object of class `mlteqc` with components

A	A equating coefficients.
B	B equating coefficients.
se.A	standard errors of A equating coefficients.
se.B	standard errors of B equating coefficients.
varAB	covariance matrix of equating coefficients.
as	discrimination parameters on a common scale $\hat{a}_j^*$ .
bs	difficulty parameters on a common scale $\hat{b}_j^*$ .
se.as	standard errors of discrimination parameters on a common scale.
se.bs	standard errors of difficulty parameters on a common scale.
tab	data frame containing item parameter names ( <code>Item</code> ), item parameter estimates of all the forms (e.g. T1, ..., T3), and item parameter estimates of all the forms converted in the scale of the base form (e.g. T3.as.T1).
varFull	list of covariance matrices of the item parameter estimates of every form.

partial	partial derivatives of equating coefficients with respect to the item parameters.
itmp	number of item parameters of the IRT model.
method	the equating method used.
basename	the name of the base form.
convergence	An integer code. 0 indicates successful convergence. Returned only with methods "irf", "trf" and "lik".

### Author(s)

Michela Battauz, Waldir Leoncio [ctb]

### References

- Battauz, M. (2017). Multiple equating of separate IRT calibrations. *Psychometrika*, **82**, 610–636.
- Battauz, M., Leoncio, W. (2023). A Likelihood Approach to Item Response Theory Equating of Multiple Forms *Applied Psychological Measurement*, **47**, 200-220. doi: 10.1177/01466216231151702.
- Haberman, S. J. (2009). Linking parameter estimates derived from an item response model through separate calibrations. ETS Research Report Series, 2009, i-9.

### See Also

[score.mlteqc](#), [modIRT](#)

### Examples

```
data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)
# Estimation of the equating coefficients with the multiple mean-mean method
eqMM <- multiec(mods = mods, base = 1, method = "mean-mean")
summary(eqMM)

# Estimation of the equating coefficients with the
# multiple mean-geometric mean method (Haberman, 2009)
eqMGM <- multiec(mods = mods, base = 1, method = "mean-gmean")
summary(eqMGM)

# Estimation of the equating coefficients with the multiple item response function method
eqIRF <- multiec(mods = mods, base = 1, method = "irf")
summary(eqIRF)

# Estimation of the equating coefficients with the multiple item response function method
# using as initial values the estimates obtained with the multiple mean-geometric mean method
eqMGM <- multiec(mods = mods, base = 1, method = "mean-gmean", se = FALSE)
eqIRF <- multiec(mods = mods, base = 1, method = "irf", start = eqMGM)
summary(eqIRF)

# Estimation of the equating coefficients with the multiple test response function method
eqTRF <- multiec(mods = mods, base = 1, method = "trf")
summary(eqTRF)
```



```
# Estimation of the equating coefficients with the likelihood-based method
eqLIK <- multiec(mods = mods, base = 1, method = "lik")
summary(eqLIK)
```

---

plot.mlteqc

*Plot of Item Parameter Estimates*


---

## Description

plot method for class mlteqc, which plots item parameter estimates of one form against the item parameter estimates of the base form before and after conversion.

## Usage

```
## S3 method for class 'mlteqc'
plot(x, form = 2, ask = prod(par("mfc01")) < x$tmp*2 && dev.interactive(), ...)
```

## Arguments

x	an object of the class mlteqc returned by function <a href="#">multiec</a> .
form	test form. It can be specified by name or number.
ask	logical; if TRUE, the user is asked before each plot.
...	further arguments passed to or from other methods.

## Author(s)

Michela Battauz

## See Also

[multiec](#)

## Examples

```
data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)
# Estimation of the equating coefficients with the multiple mean-mean method
eqMM <- multiec(mods = mods, base = 1, method = "mean-mean")
plot(eqMM, form = "T2")
```

---

 score.mlteqc

*Scoring of multiple forms*


---

### Description

Relates number-correct scores on multiple forms.

### Usage

```
## S3 method for class 'mlteqc'
score(obj, method="TSE", D=1, scores=NULL, se=TRUE, nq=30,
      w=0.5, theta=NULL, weights=NULL, ...)
```

### Arguments

obj	object of the class mlteqc returned by function <a href="#">multiec</a> .
method	the scoring method to be used. This should be one of "TSE" (the default) for true score equating or "OSE" for observed score equating.
D	constant D of the IRT model used to estimate item parameters.
scores	integer values to be converted.
se	logical; is TRUE standard errors of equated scores are computed.
nq	number of quadrature points used to approximate integrals with observed score equating. Used only if arguments theta and weights are NULL.
w	synthetic weight for population 1. It should be a number between 0 and 1.
theta	vector of ability values used to approximate integrals with observed score equating.
weights	vector of weights used to approximate integrals with observed score equating.
...	further arguments passed to or from other methods.

### Details

In this function common items are internal, i.e. they are used for scoring the test.

### Value

A data frame containing theta values (only for true score equating), scores of the form chosen as base, equated scores of all other forms, and standard errors of equated scores.

### Author(s)

Michela Battauz

## References

- Kolen, M.J. and Brennan, R.L. (2014). *Test equating, scaling, and linking: methods and practices*, 3rd ed., New York: Springer.
- Ogasawara, H. (2001). Item response theory true score equatings and their standard errors. *Journal of Educational and Behavioral Statistics*, **26**, 31–50.
- Ogasawara, H. (2003). Asymptotic standard errors of IRT observed-score equating methods. *Psychometrika*, **68**, 193–211.

## See Also

[multiec](#)

## Examples

```
data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)

# Estimation of the equating coefficients with the multiple item response function method
eqIRF<-multiec(mods = mods, base = 1, method = "irf")
summary(eqIRF)

# scoring using the true score equating method
score(eqIRF)

# scoring using observed score equating method, without standard errors
score(eqIRF, method = "OSE", se = FALSE)
```

---

summary.mlteqc

*Summarizing Estimated Equating Coefficients*

---

## Description

summary method for class mlteqc.

## Usage

```
## S3 method for class 'mlteqc'
summary(object, ...)
```

## Arguments

object            an object of the class mlteqc returned by function [multiec](#).

...                further arguments passed to or from other methods.

## Author(s)

Michela Battauz

**See Also**[multiec](#)**Examples**

```
data(est2pl)
# prepare the data
mods <- modIRT(coef = est2pl$coef, var = est2pl$var, display = FALSE)
# Estimation of the equating coefficients with the multiple mean-mean method
eqMM <- multiec(mods = mods, base = 1, method = "mean-mean")
summary(eqMM)
```

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