

# Package ‘MomTrunc’

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**Type** Package

**Title** Moments of Folded and Doubly Truncated Multivariate Distributions

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**Description** It computes arbitrary products moments (mean vector and variance-covariance matrix), for some double truncated (and folded) multivariate distributions. These distributions belong to the family of selection elliptical distributions, which includes well known skewed distributions as the unified skew-t distribution (SUT) and its particular cases as the extended skew-t (EST), skew-t (ST) and the symmetric student-t (T) distribution. Analogous normal cases unified skew-normal (SUN), extended skew-normal (ESN), skew-normal (SN), and symmetric normal (N) are also included. Density, probabilities and random deviates are also offered for these members.

**License** GPL (>= 2)

**Depends** R (>= 3.6.0)

**Imports** Rcpp (>= 1.0.1), mvtnorm (>= 1.0.11), tlmvnmvt (>= 1.1.0),  
hypergeo

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**Suggests** tmvtnorm

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 MomTrunc-package

*Moments of Folded and Doubly Truncated Multivariate Distributions*


---

## Description

It computes arbitrary products moments (mean vector and variance-covariance matrix), for some double truncated (and folded) multivariate distributions. These distributions belong to the family of selection elliptical distributions, which includes well known skewed distributions as the unified skew-t distribution (SUT) and its particular cases as the extended skew-t (EST), skew-t (ST) and the symmetric student-t (T) distribution. Analogous normal cases unified skew-normal (SUN), extended skew-normal (ESN), skew-normal (SN), and symmetric normal (N) are also included. Density, probabilities and random deviates are also offered for these members.

## Details

Probabilities can be computed using the functions `pmvSN` and `pmvESN` for the normal cases SN and ESN and, `pmvST` and `pmvEST` for the t cases ST and EST respectively, which offer the option to return the logarithm in base 2 of the probability, useful when the true probability is too small for the machine precision. These functions above use methods in Genz (1992) through the `mvtnorm` package (linked directly to our C++ functions) and Cao et.al. (2019) through the package `tlrmvnmvt`. For the double truncated Student-t cases SUT, EST, ST and T, decimal degrees of freedom are supported. Computation of arbitrary moments are based in the works of Kan & Robotti (2017) and Galarza et.al. (2021,2022a,2022b). Reference for the family of selection-elliptical distributions in this package can be found in Arellano-Valle & Genton (2005).

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## References

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- Cao, J., Genton, M. G., Keyes, D. E., & Turkiyyah, G. M. (2019) "Exploiting Low Rank Covariance Structures for Computing High-Dimensional Normal and Student-t Probabilities" <<https://marcgenton.github.io/2019.CGKT.manuscript.pdf>>.
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- Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.
- Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.
- Genz, A., "Numerical computation of multivariate normal probabilities," *Journal of Computational and Graphical Statistics*, 1, 141-149 (1992) <doi:10.1080/10618600.1992.10477010>.
- Kan, R., & Robotti, C. (2017). On moments of folded and truncated multivariate normal distributions. *Journal of Computational and Graphical Statistics*, 26(4), 930-934.

## See Also

[onlymeanTMD](#), [meanvarTMD](#), [momentsTMD](#), [dmvSN](#), [pmvSN](#), [rmvSN](#), [dmvST](#), [pmvST](#), [rmvST](#)

## Examples

```
a = c(-0.8, -0.7, -0.6)
b = c(0.5, 0.6, 0.7)
mu = c(0.1, 0.2, 0.3)
Sigma = matrix(data = c(1, 0.2, 0.3, 0.2, 1, 0.4, 0.3, 0.4, 1),
               nrow = length(mu), ncol = length(mu), byrow = TRUE)

meanvarTMD(a,b,mu,Sigma,dist="normal") #normal case
meanvarTMD(mu = mu, Sigma = Sigma, lambda = c(-2, 0, 1), dist="SN") #skew normal with NO truncation
meanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),nu = 4.87,dist = "ST") #skew t
momentsTMD(3,a,b,mu,Sigma,nu = 4,dist = "t") #t case, all moments or order <=3
```

---

cdfFMD

*Cumulative distribution function for folded multivariate distributions*

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## Description

It computes the cumulative distribution function on  $x$  for a folded  $p$ -variate Normal, Skew-normal (SN), Extended Skew-normal (ESN) and Student's  $t$ -distribution.

**Usage**

```
cdfFMD(x,mu,Sigma,lambda = NULL,tau = NULL,dist,nu = NULL)
```

**Arguments**

<code>x</code>	vector of length $p$ where the cdf is evaluated.
<code>mu</code>	a numeric vector of length $p$ representing the location parameter.
<code>Sigma</code>	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
<code>lambda</code>	a numeric vector of length $p$ representing the skewness parameter for SN and ESN cases. If <code>lambda == 0</code> , the ESN/SN reduces to a normal (symmetric) distribution.
<code>tau</code>	It represents the extension parameter for the ESN distribution. If <code>tau == 0</code> , the ESN reduces to a SN distribution.
<code>dist</code>	represents the folded distribution to be computed. The values are <code>normal</code> , <code>SN</code> , <code>ESN</code> and <code>t</code> for the doubly truncated Normal, Skew-normal, Extended Skew-normal and Student's t-distribution respectively.
<code>nu</code>	It represents the degrees of freedom for the Student's t-distribution.

**Details**

Normal case by default, i.e., when `dist` is not provided. Univariate case is also considered, where `Sigma` will be the variance  $\sigma^2$ .

**Value**

It returns the distribution value for a single point `x`.

**Note**

Degrees of freedom must be a positive integer. If `nu >= 200`, Normal case is considered."

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**References**

- Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850 <doi:10.1007/s00184-020-00802-1>.
- Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.
- Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.

**See Also**

[momentsFMD](#), [meanvarFMD](#)

**Examples**

```
mu = c(0.1,0.2,0.3,0.4)
Sigma = matrix(data = c(1,0.2,0.3,0.1,0.2,1,0.4,-0.1,0.3,0.4,1,0.2,0.1,-0.1,0.2,1),
               nrow = length(mu),ncol = length(mu),byrow = TRUE)
cdfFMD(x = c(0.5,0.2,1.0,1.3),mu,Sigma,dist="normal")
cdfFMD(x = c(0.5,0.2,1.0,1.3),mu,Sigma,dist = "t",nu = 4)
cdfFMD(x = c(0.5,0.2,1.0,1.3),mu,Sigma,lambda = c(-2,0,2,1),dist = "SN")
cdfFMD(x = c(0.5,0.2,1.0,1.3),mu,Sigma,lambda = c(-2,0,2,1),tau = 1,dist = "ESN")
```

---

dprmvESN

---

*Multivariate Extended-Skew Normal Density, Probabilities and Random Deviates Generator*


---

**Description**

These functions provide the density function, probabilities and a random number generator for the multivariate extended-skew normal (ESN) distribution with mean vector  $\mu$ , scale matrix  $\Sigma$ , skewness parameter  $\lambda$  and extension parameter  $\tau$ .

**Usage**

```
dmvESN(x,mu=rep(0,length(lambda)),Sigma=diag(length(lambda)),lambda,tau=0)
pmvESN(lower = rep(-Inf,length(lambda)),upper=rep(Inf,length(lambda)),
        mu = rep(0,length(lambda)),Sigma,lambda,tau,log2 = FALSE)
rmvESN(n,mu=rep(0,length(lambda)),Sigma=diag(length(lambda)),lambda,tau=0)
```

**Arguments**

<code>x</code>	vector or matrix of quantiles. If <code>x</code> is a matrix, each row is taken to be a quantile.
<code>n</code>	number of observations.
<code>lower</code>	the vector of lower limits of length $p$ .
<code>upper</code>	the vector of upper limits of length $p$ .
<code>mu</code>	a numeric vector of length $p$ representing the location parameter.
<code>Sigma</code>	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
<code>lambda</code>	a numeric vector of length $p$ representing the skewness parameter for SN and ESN cases. If <code>lambda == 0</code> , the ESN/SN reduces to a normal (symmetric) distribution.
<code>tau</code>	It represents the extension parameter for the ESN distribution. If <code>tau == 0</code> , the ESN reduces to a SN distribution.
<code>log2</code>	a boolean variable, indicating if the <code>log2</code> result should be returned. This is useful when the true probability is too small for the machine precision.

**Value**

dmvESN gives the density, pmvESN gives the distribution function, and rmvESN generates random deviates for the Multivariate Extended-Skew Normal Distribution.

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**References**

- Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850 <doi:10.1007/s00184-020-00802-1>.
- Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.
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- Genz, A., (1992) "Numerical computation of multivariate normal probabilities," *Journal of Computational and Graphical Statistics*, 1, 141-149 <doi:10.1080/10618600.1992.10477010>.

**See Also**

[dmvSN](#), [pmvSN](#), [rmvSN](#), [meanvarFMD](#), [meanvarTMD](#), [momentsTMD](#)

**Examples**

```
#Univariate case
dmvESN(x = -1, mu = 2, Sigma = 5, lambda = -2, tau = 0.5)
rmvESN(n = 100, mu = 2, Sigma = 5, lambda = -2, tau = 0.5)
#Multivariate case
mu = c(0.1, 0.2, 0.3, 0.4)
Sigma = matrix(data = c(1, 0.2, 0.3, 0.1, 0.2, 1, 0.4, -0.1, 0.3, 0.4, 1, 0.2, 0.1, -0.1, 0.2, 1),
               nrow = length(mu), ncol = length(mu), byrow = TRUE)
lambda = c(-2, 0, 1, 2)
tau = 2
#One observation
dmvESN(x = c(-2, -1, 0, 1), mu, Sigma, lambda, tau)
rmvESN(n = 100, mu, Sigma, lambda, tau)
#Many observations as matrix
x = matrix(rnorm(4*10), ncol = 4, byrow = TRUE)
dmvESN(x = x, mu, Sigma, lambda, tau)

lower = rep(-Inf, 4)
```

```
upper = c(-1,0,2,5)
pmvESN(lower, upper, mu, Sigma, lambda, tau)
```

---

dprmvEST	<i>Multivariate Extended-Skew t Density, Probabilities and Random Deviates Generator</i>
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## Description

These functions provide the density function, probabilities and a random number generator for the multivariate extended-skew t (EST) distribution with mean vector  $\mu$ , scale matrix  $\Sigma$ , skewness parameter  $\lambda$ , extension parameter  $\tau$  and degrees of freedom  $\nu$ .

## Usage

```
dmvEST(x, mu=rep(0, length(lambda)), Sigma=diag(length(lambda)), lambda, tau=0, nu)
pmvEST(lower = rep(-Inf, length(lambda)), upper=rep(Inf, length(lambda)),
        mu = rep(0, length(lambda)), Sigma, lambda, tau, nu, log2 = FALSE)
rmvEST(n, mu=rep(0, length(lambda)), Sigma=diag(length(lambda)), lambda, tau, nu)
```

## Arguments

$x$	vector or matrix of quantiles. If $x$ is a matrix, each row is taken to be a quantile.
$n$	number of observations.
lower	the vector of lower limits of length $p$ .
upper	the vector of upper limits of length $p$ .
$\mu$	a numeric vector of length $p$ representing the location parameter.
$\Sigma$	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
$\lambda$	a numeric vector of length $p$ representing the skewness parameter for ST and EST cases. If $\lambda == 0$ , the EST/ST reduces to a t (symmetric) distribution.
$\tau$	It represents the extension parameter for the EST distribution. If $\tau == 0$ , the EST reduces to a ST distribution.
$\nu$	It represents the degrees of freedom of the Student's t-distribution.
log2	a boolean variable, indicating if the log2 result should be returned. This is useful when the true probability is too small for the machine precision.

## Value

dmvEST gives the density, pmvEST gives the distribution function, and rmvEST generates random deviates for the Multivariate Extended-Skew- $t$  Distribution.

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## References

Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850 <doi:10.1007/s00184-020-00802-1>.

Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.

Genz, A., (1992) "Numerical computation of multivariate normal probabilities," *Journal of Computational and Graphical Statistics*, 1, 141-149 <doi:10.1080/10618600.1992.10477010>.

## See Also

[dmvST](#), [pmvST](#), [rmvST](#), [meanvarFMD](#), [meanvarTMD](#), [momentsTMD](#)

## Examples

```
#Univariate case
dmvEST(x = -1,mu = 2,Sigma = 5,lambda = -2,tau = 0.5,nu=4)
rmvEST(n = 100,mu = 2,Sigma = 5,lambda = -2,tau = 0.5,nu=4)
#Multivariate case
mu = c(0.1,0.2,0.3,0.4)
Sigma = matrix(data = c(1,0.2,0.3,0.1,0.2,1,0.4,-0.1,0.3,0.4,1,0.2,0.1,-0.1,0.2,1),
               nrow = length(mu),ncol = length(mu),byrow = TRUE)
lambda = c(-2,0,1,2)
tau = 2
#One observation
dmvEST(x = c(-2,-1,0,1),mu,Sigma,lambda,tau,nu=4)
rmvEST(n = 100,mu,Sigma,lambda,tau,nu=4)
#Many observations as matrix
x = matrix(rnorm(4*10),ncol = 4,byrow = TRUE)
dmvEST(x = x,mu,Sigma,lambda,tau,nu=4)

lower = rep(-Inf,4)
upper = c(-1,0,2,5)
pmvEST(lower,upper,mu,Sigma,lambda,tau,nu=4)
```

---

dprmvSN

*Multivariate Skew Normal Density and Probabilities and Random Deviates*

---

## Description

These functions provide the density function and a random number generator for the multivariate skew normal (SN) distribution with mean vector  $\mu$ , scale matrix  $\Sigma$  and skewness parameter  $\lambda$ .



**Usage**

```
dmvSN(x,mu=rep(0,length(lambda)),Sigma=diag(length(lambda)),lambda)
pmvSN(lower = rep(-Inf,length(lambda)),upper=rep(Inf,length(lambda)),
      mu = rep(0,length(lambda)),Sigma,lambda,log2 = FALSE)
rmvSN(n,mu=rep(0,length(lambda)),Sigma=diag(length(lambda)),lambda)
```

**Arguments**

x	vector or matrix of quantiles. If x is a matrix, each row is taken to be a quantile.
n	number of observations.
lower	the vector of lower limits of length $p$ .
upper	the vector of upper limits of length $p$ .
mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric vector of length $p$ representing the skewness parameter for SN and SN cases. If $\lambda = 0$ , the SN/SN reduces to a normal (symmetric) distribution.
log2	a boolean variable, indicating if the log2 result should be returned. This is useful when the true probability is too small for the machine precision.

**Value**

dmvSN gives the density, pmvSN gives the distribution function, and rmvSN generates random deviates for the Multivariate Skew-normal Distribution.

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**References**

- Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.
- Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.
- Galarza, C.E., Matos, L.A. and Lachos, V.H. (2022c). An EM algorithm for estimating the parameters of the multivariate skew-normal distribution with censored responses. *Metron*. <doi:10.1007/s40300-021-00227-4>.
- Genz, A., (1992) "Numerical computation of multivariate normal probabilities," *Journal of Computational and Graphical Statistics*, 1, 141-149 <doi:10.1080/10618600.1992.10477010>.

**See Also**

[dmvESN](#), [pmvESN](#), [rmvESN](#), [meanvarFMD](#), [meanvarTMD](#), [momentsTMD](#)

**Examples**

```
#Univariate case
dmvSN(x = -1,mu = 2,Sigma = 5,lambda = -2)
rmvSN(n = 100,mu = 2,Sigma = 5,lambda = -2)
#Multivariate case
mu = c(0.1,0.2,0.3,0.4)
Sigma = matrix(data = c(1,0.2,0.3,0.1,0.2,1,0.4,-0.1,0.3,0.4,1,0.2,0.1,-0.1,0.2,1),
               nrow = length(mu),ncol = length(mu),byrow = TRUE)
lambda = c(-2,0,1,2)
#One observation
dmvSN(x = c(-2,-1,0,1),mu,Sigma,lambda)
rmvSN(n = 100,mu,Sigma,lambda)
#Many observations as matrix
x = matrix(rnorm(4*10),ncol = 4,byrow = TRUE)
dmvSN(x = x,mu,Sigma,lambda)

lower = rep(-Inf,4)
upper = c(-1,0,2,5)
pmvSN(lower,upper,mu,Sigma,lambda)
```

---

dprmvST

*Multivariate Skew t Density, Probabilities and Random Deviates Generator*

---

**Description**

These functions provide the density function, probabilities and a random number generator for the multivariate skew t (EST) distribution with mean vector  $\mu$ , scale matrix  $\Sigma$ , skewness parameter  $\lambda$  and degrees of freedom  $\nu$ .

**Usage**

```
dmvST(x,mu=rep(0,length(lambda)),Sigma=diag(length(lambda)),lambda,nu)
pmvST(lower = rep(-Inf,length(lambda)),upper=rep(Inf,length(lambda)),
      mu = rep(0,length(lambda)),Sigma,lambda,nu,log2 = FALSE)
rmvST(n,mu=rep(0,length(lambda)),Sigma=diag(length(lambda)),lambda,nu)
```

**Arguments**

$x$  vector or matrix of quantiles. If  $x$  is a matrix, each row is taken to be a quantile.  
 $n$  number of observations.  
 $lower$  the vector of lower limits of length  $p$ .  
 $upper$  the vector of upper limits of length  $p$ .

mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric vector of length $p$ representing the skewness parameter for ST and EST cases. If $\text{lambda} == 0$ , the EST/ST reduces to a t (symmetric) distribution.
nu	It represents the degrees of freedom of the Student's t-distribution.
log2	a boolean variable, indicating if the log2 result should be returned. This is useful when the true probability is too small for the machine precision.

### Value

dmvST gives the density, pmvST gives the distribution function, and rmvST generates random deviates for the Multivariate Skew- $t$  Distribution.

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### References

- Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850 <doi:10.1007/s00184-020-00802-1>.
- Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.
- Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.
- Genz, A., (1992) "Numerical computation of multivariate normal probabilities," *Journal of Computational and Graphical Statistics*, 1, 141-149 <doi:10.1080/10618600.1992.10477010>.

### See Also

[dmvST](#), [pmvST](#), [rmvST](#), [meanvarFMD](#), [meanvarTMD](#), [momentsTMD](#)

### Examples

```
#Univariate case
dmvST(x = -1,mu = 2,Sigma = 5,lambda = -2,nu=4)
rmvST(n = 100,mu = 2,Sigma = 5,lambda = -2,nu=4)
#Multivariate case
mu = c(0.1,0.2,0.3,0.4)
Sigma = matrix(data = c(1,0.2,0.3,0.1,0.2,1,0.4,-0.1,0.3,0.4,1,0.2,0.1,-0.1,0.2,1),
               nrow = length(mu),ncol = length(mu),byrow = TRUE)
lambda = c(-2,0,1,2)
#One observation
```

```

dmvST(x = c(-2,-1,0,1),mu,Sigma,lambda,nu=4)
rmvST(n = 100,mu,Sigma,lambda,nu=4)
#Many observations as matrix
x = matrix(rnorm(4*10),ncol = 4,byrow = TRUE)
dmvST(x = x,mu,Sigma,lambda,nu=4)

lower = rep(-Inf,4)
upper = c(-1,0,2,5)
pmvST(lower,upper,mu,Sigma,lambda,nu=4)

```

---

MCmeanvarTMD

*Monte Carlo Mean and variance for doubly truncated multivariate distributions*


---

### Description

It computes the Monte Carlo mean vector and variance-covariance matrix for some doubly truncated skew-elliptical distributions. Monte Carlo simulations are performed via slice Sampling. It supports the p-variate Normal, Skew-normal (SN), Extended Skew-normal (ESN) and Unified Skew-normal (SUN) as well as the Student's-t, Skew-t (ST), Extended Skew-t (EST) and Unified Skew-t (SUT) distribution.

### Usage

```

MCmeanvarTMD(lower = rep(-Inf,length(mu)),upper = rep(Inf,length(mu)),mu,Sigma
,lambda = NULL,tau = NULL,Gamma = NULL,nu = NULL,dist,n = 10000)

```

### Arguments

lower	the vector of lower limits of length $p$ .
upper	the vector of upper limits of length $p$ .
mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric matrix of dimension $p \times q$ representing the skewness/shape matrix parameter for the SUN and SUT distribution. For the ESN and EST distributions ( $q = 1$ ), lambda is a numeric vector of dimension $p$ (see examples at the end of this help). If all(lambda == 0), the SUN/ESN/SN (SUT/EST/ST) reduces to a normal (t) symmetric distribution.
tau	a numeric vector of length $q$ representing the extension parameter for the SUN and SUT distribution. For the ESN and EST distributions, tau is a positive scalar ( $q = 1$ ). Furthermore, if tau == 0, the ESN (EST) reduces to a SN (ST) distribution.
Gamma	a correlation matrix with dimension $q \times q$ . It must be provided only for the SUN and SUT cases. For particular cases SN, ESN, ST and EST, we have that Gamma == 1 (see examples at the end of this help).

nu	It represents the degrees of freedom for the Student's t-distribution being a positive real number.
dist	represents the truncated distribution to be used. The values are normal, SN , ESN and SUN for the doubly truncated Normal, Skew-normal, Extended Skew-normal and Unified-skew normal distributions and, t, ST , EST and SUT for the doubly truncated Student-t, Skew-t, Extended Skew-t and Unified skew-t distributions.
n	number of Monte Carlo samples to be generated.

**Value**

It returns a list with three elements:

mean	the estimate for the mean vector of length $p$
EYY	the estimate for the second moment matrix of dimensions $p \times p$
varcov	the estimate for the variance-covariance matrix of dimensions $p \times p$

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**References**

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Ho, H. J., Lin, T. I., Chen, H. Y., & Wang, W. L. (2012). Some results on the truncated multivariate t distribution. *Journal of Statistical Planning and Inference*, 142(1), 25-40.

**See Also**

[meanvarTMD](#), [rmvSN](#), [rmvESN](#), [rmvST](#), [rmvEST](#)

**Examples**

```
a = c(-0.8, -0.7, -0.6)
b = c(0.5, 0.6, 0.7)
mu = c(0.1, 0.2, 0.3)
Sigma = matrix(data = c(1, 0.2, 0.3, 0.2, 1, 0.4, 0.3, 0.4, 1),
               nrow = length(mu), ncol = length(mu), byrow = TRUE)

## Normal case

# Theoretical value
value1 = meanvarTMD(a,b,mu,Sigma,dist="normal")

#MC estimate
MC11 = MCmeanvarTMD(a,b,mu,Sigma,dist="normal") #by defalut n = 10000
```

```

MC12 = MCmeanvarTMD(a,b,mu,Sigma,dist="normal",n = 10^5) #more precision

## Skew-t case

# Theoretical value
value2 = meanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),nu = 4,dist = "ST")

#MC estimate
MC21 = MCmeanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),nu = 4,dist = "ST")

## More...

MC5 = MCmeanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),tau = 1,dist = "ESN")
MC6 = MCmeanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),tau = 1,nu = 4,dist = "EST")

#Skew-unified Normal (SUN) and Skew-unified t (SUT) distributions

Lambda = matrix(c(1,0,2,-3,0,-1),3,2) #A skewness matrix p times q
Gamma = matrix(c(1,-0.5,-0.5,1),2,2) #A correlation matrix q times q
tau = c(-1,2) #A vector of extension parameters of dim q

MC7 = MCmeanvarTMD(a,b,mu,Sigma,lambda = Lambda,tau = c(-1,2),Gamma = Gamma,dist = "SUN")
MC8 = MCmeanvarTMD(a,b,mu,Sigma,lambda = Lambda,tau = c(-1,2),Gamma = Gamma,nu = 1,dist = "SUT")

```

---

meanvarFMD

*Mean and variance for folded multivariate distributions*

---

## Description

It computes the mean vector and variance-covariance matrix for the folded  $p$ -variate Normal, Skew-normal (SN), Extended Skew-normal (ESN) and Student's  $t$ -distribution.

## Usage

```
meanvarFMD(mu,Sigma,lambda = NULL,tau = NULL,nu = NULL,dist)
```

## Arguments

mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric vector of length $p$ representing the skewness parameter for SN and ESN cases. If $\lambda = 0$ , the ESN/SN reduces to a normal (symmetric) distribution.
tau	It represents the extension parameter for the ESN distribution. If $\tau = 0$ , the ESN reduces to a SN distribution.

nu	It represents the degrees of freedom for the Student's t-distribution. Must be an integer greater than 1.
dist	represents the folded distribution to be computed. The values are normal, SN, ESN and t for the doubly truncated Normal, Skew-normal, Extended Skew-normal and Student's t-distribution respectively.

### Details

Normal case by default, i.e., when dist is not provided. Univariate case is also considered, where Sigma will be the variance  $\sigma^2$ .

### Value

It returns a list with three elements:

mean	the mean vector of length $p$
EYY	the second moment matrix of dimensions $pxp$
varcov	the variance-covariance matrix of dimensions $pxp$

### Warning

The mean can only be provided when nu is larger than 2. On the other hand, the varcov matrix can only be provided when nu is larger than 3.

### Note

Degree of freedom must be a positive integer. If nu  $\geq 200$ , Normal case is considered."

### Author(s)

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### References

- Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850 <doi:10.1007/s00184-020-00802-1>.
- Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.
- Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.

### See Also

[momentsFMD](#), [onlymeanTMD](#), [meanvarTMD](#), [momentsTMD](#), [dmvSN](#), [pmvSN](#), [rmvSN](#), [dmvESN](#), [pmvESN](#), [rmvESN](#), [dmvST](#), [pmvST](#), [rmvST](#), [dmvEST](#), [pmvEST](#), [rmvEST](#)

**Examples**

```

mu = c(0.1,0.2,0.3)
Sigma = matrix(data = c(1,0.2,0.3,0.2,1,0.4,0.3,0.4,1),
               nrow = length(mu),ncol = length(mu),byrow = TRUE)
value1 = meanvarFMD(mu,Sigma,dist="normal")
value2 = meanvarFMD(mu,Sigma,nu = 4,dist = "t")
value3 = meanvarFMD(mu,Sigma,lambda = c(-2,0,1),dist = "SN")
value4 = meanvarFMD(mu,Sigma,lambda = c(-2,0,1),tau = 1,dist = "ESN")

```

---

meanvarTMD

---

*Mean and variance for doubly truncated multivariate distributions*


---

**Description**

It computes the mean vector and variance-covariance matrix for some doubly truncated skew-elliptical distributions. It supports the  $p$ -variate Normal, Skew-normal (SN), Extended Skew-normal (ESN) and Unified Skew-normal (SUN) as well as the Student's-t, Skew-t (ST), Extended Skew-t (EST) and Unified Skew-t (SUT) distribution.

**Usage**

```

meanvarTMD(lower = rep(-Inf, length(mu)), upper = rep(Inf, length(mu)), mu, Sigma
, lambda = NULL, tau = NULL, Gamma = NULL, nu = NULL, dist)

```

**Arguments**

lower	the vector of lower limits of length $p$ .
upper	the vector of upper limits of length $p$ .
mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric matrix of dimension $p \times q$ representing the skewness/shape matrix parameter for the SUN and SUT distribution. For the ESN and EST distributions ( $q = 1$ ), lambda is a numeric vector of dimension $p$ (see examples at the end of this help). If <code>all(lambda == 0)</code> , the SUN/ESN/SN (SUT/EST/ST) reduces to a normal (t) symmetric distribution.
tau	a numeric vector of length $q$ representing the extension parameter for the SUN and SUT distribution. For the ESN and EST distributions, tau is a positive scalar ( $q = 1$ ). Furthermore, if <code>tau == 0</code> , the ESN (EST) reduces to a SN (ST) distribution.
Gamma	a correlation matrix with dimension $q \times q$ . It must be provided only for the SUN and SUT cases. For particular cases SN, ESN, ST and EST, we have that <code>Gamma == 1</code> (see examples at the end of this help).
nu	It represents the degrees of freedom for the Student's t-distribution being a positive real number.



`dist` represents the truncated distribution to be used. The values are normal, SN, ESN and SUN for the doubly truncated Normal, Skew-normal, Extended Skew-normal and Unified-skew normal distributions and, `t`, ST, EST and SUT for the doubly truncated Student-t, Skew-t, Extended Skew-t and Unified skew-t distributions.

### Details

Univariate case is also considered, where Sigma will be the variance  $\sigma^2$ . Normal case code is an R adaptation of the Matlab available function `dtmvnmom.m` from Kan & Robotti (2017) and it is used for  $p \leq 3$ . For higher dimensions we use an extension of the algorithm in Vaida (2009).

### Value

It returns a list with three elements:

<code>mean</code>	the mean vector of length $p$
<code>EYY</code>	the second moment matrix of dimensions $p \times p$
<code>varcov</code>	the variance-covariance matrix of dimensions $p \times p$

### Warning

For the  $t$  cases, the algorithm supports degrees of freedom  $\nu \leq 2$ .

### Note

If  $\nu \geq 300$ , Normal case is considered."

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### References

- Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850 <doi:10.1007/s00184-020-00802-1>.
- Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.
- Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.

### See Also

[MCmeanvarTMD](#), [momentsTMD](#), [meanvarFMD](#), [meanvarFMD,momentsFMD](#), [dmvSN](#), [pmvSN](#), [rmvSN](#), [dmvESN](#), [pmvESN](#), [rmvESN](#), [dmvST](#), [pmvST](#), [rmvST](#), [dmvEST](#), [pmvEST](#), [rmvEST](#)

**Examples**

```

a = c(-0.8,-0.7,-0.6)
b = c(0.5,0.6,0.7)
mu = c(0.1,0.2,0.3)
Sigma = matrix(data = c(1,0.2,0.3,0.2,1,0.4,0.3,0.4,1),
               nrow = length(mu),ncol = length(mu),byrow = TRUE)

# Theoretical value
value1 = meanvarTMD(a,b,mu,Sigma,dist="normal")

#MC estimate
MC11 = MCmeanvarTMD(a,b,mu,Sigma,dist="normal") #by default n = 10000
MC12 = MCmeanvarTMD(a,b,mu,Sigma,dist="normal",n = 10^5) #more precision

# Now works for any nu>0
value2 = meanvarTMD(a,b,mu,Sigma,dist = "t",nu = 0.87)

value3 = meanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),dist = "SN")
value4 = meanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),nu = 4,dist = "ST")
value5 = meanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),tau = 1,dist = "ESN")
value6 = meanvarTMD(a,b,mu,Sigma,lambda = c(-2,0,1),tau = 1,nu = 4,dist = "EST")

#Skew-unified Normal (SUN) and Skew-unified t (SUT) distributions

Lambda = matrix(c(1,0,2,-3,0,-1),3,2) #A skewness matrix p times q
Gamma = matrix(c(1,-0.5,-0.5,1),2,2) #A correlation matrix q times q
tau = c(-1,2) #A vector of extension parameters of dim q

value7 = meanvarTMD(a,b,mu,Sigma,lambda = Lambda,tau = c(-1,2),Gamma = Gamma,dist = "SUN")
value8 = meanvarTMD(a,b,mu,Sigma,lambda = Lambda,tau = c(-1,2),Gamma = Gamma,nu = 4,dist = "SUT")

#The ESN and EST as particular cases of the SUN and SUT for q=1

Lambda = matrix(c(-2,0,1),3,1)
Gamma = 1
value9 = meanvarTMD(a,b,mu,Sigma,lambda = Lambda,tau = 1,Gamma = Gamma,dist = "SUN")
value10 = meanvarTMD(a,b,mu,Sigma,lambda = Lambda,tau = 1,Gamma = Gamma,nu = 4,dist = "SUT")

round(value5$varcov,2) == round(value9$varcov,2)
round(value6$varcov,2) == round(value10$varcov,2)

```

**Description**

It computes the kappa-th order moments for the folded p-variate Normal, Skew-normal (SN), Extended Skew-normal (ESN) and Student's t-distribution. It also outputs other lower moments involved in the recurrence approach.

**Usage**

```
momentsFMD(kappa,mu,Sigma,lambda = NULL,tau = NULL,nu = NULL,dist)
```

**Arguments**

kappa	moments vector of length $p$ . All its elements must be integers greater or equal to 0. For the Student's-t case, kappa can be a scalar representing the order of the moment.
mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric vector of length $p$ representing the skewness parameter for SN and ESN cases. If $\lambda = \emptyset$ , the ESN/SN reduces to a normal (symmetric) distribution.
tau	It represents the extension parameter for the ESN distribution. If $\tau = \emptyset$ , the ESN reduces to a SN distribution.
nu	It represents the degrees of freedom for the Student's t-distribution. Must be an integer greater than 1.
dist	represents the folded distribution to be computed. The values are normal, SN, ESN and t for the doubly truncated Normal, Skew-normal, Extended Skew-normal and Student's t-distribution respectively.

**Details**

Univariate case is also considered, where Sigma will be the variance  $\sigma^2$ .

**Value**

A data frame containing  $p + 1$  columns. The  $p$  first containing the set of combinations of exponents summing up to kappa and the last column containing the expected value. Normal cases (ESN, SN and normal) return  $\text{prod}(\text{kappa}) + 1$  moments while the Student's t-distribution case returns all moments of order up to kappa. See example section.

**Warning**

For the Student-t cases, including ST and EST, kappa-th order moments exist only for  $\text{kappa} < \text{nu}$ .

**Note**

Degrees of freedom must be a positive integer. If  $\text{nu} \geq 300$ , Normal case is considered."

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**References**

Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850 <doi:10.1007/s00184-020-00802-1>.

Galarza, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.

Galarza, C. E., Matos, L. A., Castro, L. M., & Lachos, V. H. (2022b). Moments of the doubly truncated selection elliptical distributions with emphasis on the unified multivariate skew-t distribution. *Journal of Multivariate Analysis*, 189, 104944 <doi:10.1016/j.jmva.2021.104944>.

**See Also**

[meanvarFMD](#), [onlymeanTMD](#), [meanvarTMD](#), [momentsTMD](#), [dmvSN](#), [pmvSN](#), [rmvSN](#), [dmvESN](#), [pmvESN](#), [rmvESN](#), [dmvST](#), [pmvST](#), [rmvST](#), [dmvEST](#), [pmvEST](#), [rmvEST](#)

**Examples**

```
mu = c(0.1, 0.2, 0.3)
Sigma = matrix(data = c(1, 0.2, 0.3, 0.2, 1, 0.4, 0.3, 0.4, 1),
               nrow = length(mu), ncol = length(mu), byrow = TRUE)
value1 = momentsFMD(c(2, 0, 1), mu, Sigma, dist = "normal")
value2 = momentsFMD(3, mu, Sigma, dist = "t", nu = 7)
value3 = momentsFMD(c(2, 0, 1), mu, Sigma, lambda = c(-2, 0, 1), dist = "SN")
value4 = momentsFMD(c(2, 0, 1), mu, Sigma, lambda = c(-2, 0, 1), tau = 1, dist = "ESN")

#T case with kappa vector input
value5 = momentsFMD(c(2, 0, 1), mu, Sigma, dist = "t", nu = 7)
```

---

momentsTMD

---

*Moments for doubly truncated multivariate distributions*


---

**Description**

It computes kappa-th order moments for for some doubly truncated skew-elliptical distributions. It supports the p-variate Normal, Skew-normal (SN) and Extended Skew-normal (ESN), as well as the Student's-t, Skew-t (ST) and the Extended Skew-t (EST) distribution.

**Usage**

```
momentsTMD(kappa, lower = rep(-Inf, length(mu)), upper = rep(Inf, length(mu)), mu, Sigma,
           lambda = NULL, tau = NULL, nu = NULL, dist)
```

**Arguments**

kappa	moments vector of length $p$ . All its elements must be integers greater or equal to 0. For the Student's-t case, kappa can be a scalar representing the order of the moment.
lower	the vector of lower limits of length $p$ .
upper	the vector of upper limits of length $p$ .
mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric vector of length $p$ representing the skewness parameter for SN and ESN cases. If $\lambda = 0$ , the ESN/SN reduces to a normal (symmetric) distribution.
tau	It represents the extension parameter for the ESN distribution. If $\tau = 0$ , the ESN reduces to a SN distribution.
nu	It represents the degrees of freedom for the Student's t-distribution being a positive real number.
dist	represents the truncated distribution to be used. The values are normal, SN and ESN for the doubly truncated Normal, Skew-normal and Extended Skew-normal distributions and, t, ST and EST for the for the doubly truncated Student-t, Skew-t and Extended Skew-t distributions.

**Details**

Univariate case is also considered, where Sigma will be the variance  $\sigma^2$ .

**Value**

A data frame containing  $p + 1$  columns. The  $p$  first containing the set of combinations of exponents summing up to kappa and the last column containing the the expected value. Normal cases (ESN, SN and normal) return  $\text{prod}(\text{kappa})+1$  moments while the Student's t-distribution case returns all moments of order up to kappa. See example section.

**Note**

If  $\text{nu} \geq 300$ , Normal case is considered."

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**References**

Galarza, C. E., Lin, T. I., Wang, W. L., & Lachos, V. H. (2021). On moments of folded and truncated multivariate Student-t distributions based on recurrence relations. *Metrika*, 84(6), 825-850.

Galarza-Morales, C. E., Matos, L. A., Dey, D. K., & Lachos, V. H. (2022a). "On moments of folded and doubly truncated multivariate extended skew-normal distributions." *Journal of Computational and Graphical Statistics*, 1-11 <doi:10.1080/10618600.2021.2000869>.

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Kan, R., & Robotti, C. (2017). On moments of folded and truncated multivariate normal distributions. *Journal of Computational and Graphical Statistics*, 26(4), 930-934.

### See Also

[onlymeanTMD](#), [meanvarTMD](#), [momentsFMD](#), [meanvarFMD](#), [dmvSN](#), [pmvSN](#), [rmvSN](#), [dmvESN](#), [pmvESN](#), [rmvESN](#), [dmvST](#), [pmvST](#), [rmvST](#), [dmvEST](#), [pmvEST](#), [rmvEST](#)

### Examples

```
a = c(-0.8, -0.7, -0.6)
b = c(0.5, 0.6, 0.7)
mu = c(0.1, 0.2, 0.3)
Sigma = matrix(data = c(1, 0.2, 0.3, 0.2, 1, 0.4, 0.3, 0.4, 1),
               nrow = length(mu), ncol = length(mu), byrow = TRUE)
value1 = momentsTMD(c(2, 0, 1), a, b, mu, Sigma, dist = "normal")
value2 = momentsTMD(c(2, 0, 1), a, b, mu, Sigma, dist = "t", nu = 7)
value3 = momentsTMD(c(2, 0, 1), a, b, mu, Sigma, lambda = c(-2, 0, 1), dist = "SN")
value4 = momentsTMD(c(2, 0, 1), a, b, mu, Sigma, lambda = c(-2, 0, 1), tau = 1, dist = "ESN")

#T cases with kappa scalar (all moments up to 3)
value5 = momentsTMD(3, a, b, mu, Sigma, nu = 7, dist = "t")
value6 = momentsTMD(3, a, b, mu, Sigma, lambda = c(-2, 0, 1), nu = 7, dist = "ST")
value7 = momentsTMD(3, a, b, mu, Sigma, lambda = c(-2, 0, 1), tau = 1, nu = 7, dist = "EST")
```

---

onlymeanTMD

*Mean for doubly truncated multivariate distributions*

---

### Description

It computes the mean vector for some doubly truncated skew-elliptical distributions. It supports the p-variate Normal, Skew-normal (SN), Extended Skew-normal (ESN) and Unified Skew-normal (SUN) as well as the Student's-t, Skew-t (ST), Extended Skew-t (EST) and Unified Skew-t (SUT) distribution.

### Usage

```
onlymeanTMD(lower = rep(-Inf, length(mu)), upper = rep(Inf, length(mu)), mu, Sigma,
            lambda = NULL, tau = NULL, Gamma = NULL, nu = NULL, dist)
```

**Arguments**

lower	the vector of lower limits of length $p$ .
upper	the vector of upper limits of length $p$ .
mu	a numeric vector of length $p$ representing the location parameter.
Sigma	a numeric positive definite matrix with dimension $p \times p$ representing the scale parameter.
lambda	a numeric vector of length $p$ representing the skewness parameter for SN and ESN cases. If $\lambda == 0$ , the ESN/SN reduces to a normal (symmetric) distribution.
tau	It represents the extension parameter for the ESN distribution. If $\tau == 0$ , the ESN reduces to a SN distribution.
Gamma	a correlation matrix with dimension $q \times q$ . It must be provided only for the SUN and SUT cases. For particular cases SN, ESN, ST and EST, we have that $\Gamma == 1$ (see examples at the end of this help).
nu	It represents the degrees of freedom for the Student's t-distribution.
dist	represents the truncated distribution to be used. The values are normal, SN, ESN and SUN for the doubly truncated Normal, Skew-normal, Extended Skew-normal and Unified-skew normal distributions and, t, ST, EST and SUT for the doubly truncated Student-t, Skew-t, Extended Skew-t and Unified skew-t distributions.

**Details**

Univariate case is also considered, where Sigma will be the variance  $\sigma^2$ . Normal case code is an R adaptation of the Matlab available function `dtmvnmom.m` from Kan & Robotti (2017) and it is used for  $p \leq 3$ . For higher dimensions we use proposal in Galarza (2022b).

**Value**

It returns the mean vector of length  $p$ .

**Note**

Degrees of freedom must be a positive integer. If  $\nu \geq 300$ , Normal case is considered."

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**References**

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### See Also

[momentsTMD](#), [meanvarFMD](#), [momentsFMD](#), [dmvESN](#), [rmvESN](#)

### Examples

```
a = c(-0.8, -0.7, -0.6)
b = c(0.5, 0.6, 0.7)
mu = c(0.1, 0.2, 0.3)
Sigma = matrix(data = c(1, 0.2, 0.3, 0.2, 1, 0.4, 0.3, 0.4, 1),
               nrow = length(mu), ncol = length(mu), byrow = TRUE)
value1 = onlymeanTMD(a, b, mu, Sigma, dist = "normal")

# Now works for for any nu > 0
value2 = onlymeanTMD(a, b, mu, Sigma, dist = "t", nu = 0.87)

value3 = onlymeanTMD(a, b, mu, Sigma, lambda = c(-2, 0, 1), dist = "SN")
value4 = onlymeanTMD(a, b, mu, Sigma, lambda = c(-2, 0, 1), tau = 1, dist = "ESN")
value5 = onlymeanTMD(a, b, mu, Sigma, lambda = c(-2, 0, 1), tau = 1, nu = 4, dist = "EST")

#Skew-unified Normal (SUN) and Skew-unified t (SUT) distributions

Lambda = matrix(c(1, 0, 2, -3, 0, -1), 3, 2) #A skewness matrix p times q
Gamma = matrix(c(1, -0.5, -0.5, 1), 2, 2) #A correlation matrix q times q
tau = c(-1, 2) #A vector of extension parameters of dim q

value6 = onlymeanTMD(a, b, mu, Sigma, lambda = Lambda, tau = c(-1, 2), Gamma = Gamma, dist = "SUN")
value7 = onlymeanTMD(a, b, mu, Sigma, lambda = Lambda, tau = c(-1, 2), Gamma = Gamma, nu = 4, dist = "SUT")

#The ESN and EST as particular cases of the SUN and SUT for q=1

Lambda = matrix(c(-2, 0, 1), 3, 1)
Gamma = 1
value8 = onlymeanTMD(a, b, mu, Sigma, lambda = Lambda, tau = 1, Gamma = Gamma, dist = "SUN")
value9 = onlymeanTMD(a, b, mu, Sigma, lambda = Lambda, tau = 1, Gamma = Gamma, nu = 4, dist = "SUT")

round(value4, 2) == round(value8, 2)
round(value5, 2) == round(value9, 2)
```



pvmnormt

*Multivariate normal and Student-t probabilities***Description**

Computation of Multivariate normal and Student-t probabilities using the classic Genz method from packages mvtnorm and tlmvnmvt packages. In order to save computational effort, it chooses whether to use the function pmvtnorm (pmvt) from mvtnorm, or functions pmvn (pmvt) from the tlmvnmvt package, depending of the vector size  $p$ , real or integer degrees of freedom  $\nu$ .

**Usage**

```
pvmnormt(lower = rep(-Inf,ncol(sigma)),upper = rep(Inf,ncol(sigma)),
mean = rep(0,ncol(sigma)),sigma,nu = NULL,uselog2 = FALSE)
```

**Arguments**

lower	lower integration limits, a numeric vector of length $p$
upper	upper integration limits, a numeric vector of length $p$
mean	the location parameter, a numeric vector of length $p$
sigma	the scale matrix, a square matrix that matches the length of 'lower'
nu	degrees of freedom, a positive real number. If NULL, normal case is considered
uselog2	a boolean variable, indicating if the log2 result should be returned. This is useful when the true probability is too small for the machine precision

**Value**

The estimated probability or its log2 if uselog2 == TRUE

**Note**

If `is.null(nu)`, normal case is considered.

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Cao, J., Genton, M. G., Keyes, D. E., & Turkiyyah, G. M. "Exploiting Low Rank Covariance Structures for Computing High-Dimensional Normal and Student- t Probabilities" (2019) <<https://marcgenton.github.io/2019.CGK>>

**See Also**

[onlymeanTMD](#), [meanvarTMD](#), [momentsFMD](#), [momentsTMD](#), [meanvarFMD](#), [dmvSN](#), [pmvSN](#), [rmvSN](#), [dmvESN](#), [pmvESN](#), [rmvESN](#), [dmvST](#), [pmvST](#), [rmvST](#), [dmvEST](#), [pmvEST](#), [rmvEST](#)

**Examples**

```
a = c(-0.8, -0.7, -0.6)
b = c(0.5, 0.6, 0.7)
mu = c(0.1, 0.2, 0.3)
Sigma = matrix(data = c(1, 0.2, 0.3, 0.2, 1, 0.4, 0.3, 0.4, 1),
               nrow = length(mu), ncol = length(mu), byrow = TRUE)

pmvnormt(lower = a, upper = b, mean = mu, sigma = Sigma) #normal case
pmvnormt(lower = a, upper = b, mean = mu, sigma = Sigma, nu = 4.23) #t case
pmvnormt(lower = a, upper = b, mean = mu, sigma = Sigma, nu = 4.23, uselog2 = TRUE)
```

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