Spontaneous Focusing on Numerosity is Linked to Numerosity Discrimination in Children and Adults

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Abstract

Spontaneously focus on numerosity (SFON) is the spontaneous tendency to address numerical information without any external motivation. A number of studies have found correlations between SFON and children’s mathematical abilities, but it is still unclear whether this relationship is due to SFON reflecting individual differences in quantitative processing. Therefore, this study examined the relationship between SFON tendency and numerosity discrimination ratios in children and adults. Children’s SFON was measured using a set of imitation tasks developed by Hannuella and Lehtinen (2005). To measure SFON in adults, we developed a computerized Numerosity Bias Task (NBT). In this task, quantitative information was presented alongside non-quantitative information that could compete with it, but subjects were not instructed in any way to regard the quantitative information. Discrimination ratios were measured using a dot-discrimination task. In both children and adults, a relationship between SFON and discrimination ratios was found such that individuals with better discrimination ratios were more likely to spontaneously attended to numerosity. Additionally, adults who spontaneously attended to numerosity also had higher academic mathematical achievements. These results suggest that increased acuity of the analog magnitude system (AMS) makes numerosity information more salient in the environment, therefore increasing the chances that the individual would attend to this aspect of the environment. Additionally, these results support the existence of a relationship between non-symbolic numerosity processing and symbolic-academic skills.

Keywords: spontaneous attention; attentional bias; numerosity; Weber ratio, mathematical abilities; mathematical achievements.

# Introduction

Humans have an innate and adaptive system for processing quantities (Carey, 2009; Spelke & Kinzler, 2007). This system, dubbed the *analog magnitude system* (AMS) is proposed to be a congenital system of numerical representation characterized by an imprecise ability to distinguish quantities by relying on estimation (Mou & Van Marle, 2014), which is common among human and some nonhuman animal species (Carey, 2009; Feigenson, Dehaene, & Spelke, 2004; Rugani, Vallortigara, & Regolin, 2013). The AMS distinguishes between quantities on the basis of ratio, with this discrimination ability improving as we develop, with the greatest improvement occurring in the first year of life (Brannon, Suanda, & Libertus, 2007; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Lipton & Spelke, 2004; Xu & Spelke, 2000). For example, day-old infants are able to differentiate quantities with a ratio of 1:3 (Izard, Sann, Spelke, & Streri, 2009), while at the age of 6 months the discrimination ratio threshold of infants improves to a ratio of 1:2 (Brannon et al., 2007; Xu & Spelke, 2000), and at the age of 9 months it improves to a ratio of 2:3 (Lipton & Spelke, 2003, 2004; Xu & Arriaga, 2007). By 3-4 years of age, children are already capable of discriminating ratios of 3:4, and adult humans can discriminate quantities with a ratio of 7:8, with some even succeeding with a ratio of 9:10 (Halberda & Feigenson, 2008). The AMS’ improvement in differentiation between quantities, on the bases of ratio, demonstrates Weber’s Law, which states that the discrimination threshold between two given stimuli (of any given type of sensory modality) increases by a given factor as the intensity of the stimulus grows (e.g., Jordan & Brannon, 2006).

Although the AMS plays a crucial role in human cognition, individuals differ in their resolution of quantitative processing and numerosity discrimination (Halberda & Feigenson, 2008). One of the most well-known computerized tasks used to measure individual discrimination ratios is the Panamath dot-discrimination task. During the task, participants are presented with two dot-arrays, and are asked to indicate which is more numerous, with the task becoming more difficult when the ratio between both dot-arrays is close (as a result of Weber’s Law). Differences in individual Weber ratios (ratio required for successful discrimination) estimated using this task have been found to be a domain-specific marker for mathematical abilities (Halberda & Feigenson, 2008; Halberda, Mazzocco, & Feigenson, 2008). For example, performance on the Panamath task has been found to correlate (Halberda et al., 2008) and even predict academic mathematical achievements over a 6-month period (Libertus, Feigenson, & Halberda, 2013). Moreover, children’s performance on a symbolic math task have been found to improve following numerical discrimination practice using the Panamath task (Wang, Odic, Halberda, & Feigenson, 2016), indicating a causal link between Panamath accuracy and symbolic math performance.

We are aware of the literature that criticizes tasks that investigate nonsymbolic numbers using dot arrays, claiming that it is difficult to separate changes in numerosity from simultaneous changes in the continuous visual properties of the stimulus (e.g., Gebuis & Reynvoet, 2012). On the other hand, numerosity has been demonstrated as playing a role in nonsymbolic number perception beyond the effects of continuous visual properties (e.g., Cordes & Brannon, 2008). This debate is beyond the scope of the current study. However, it is important to keep in mind that changes in numerosity (as in real life) are always accompanied by changes in other continuous perceptual properties, such as expanded item size, total surface area, density, and circumference, and hence may be seen as part of one continuous variable (Leibovich, Katzin, Harel, & Henik, 2017).

Individual differences in numerical processing have also been found in regard to the tendency to attend numerical information; Numerical information, even if present, is not always attended to, and studies have found individual differences in the tendency to address numerical information without any external motivation (Hannula & Lehtinen, 2005). Individuals who show this tendency of spontaneous focusing on numerosity (SFON) tend to attend to numerical aspects, identify numerosities, and make use of numerical information, all without any necessary external trigger, encouragement, or guidance to do so (Hannula & Lehtinen, 2005).

So far, such individual differences have been explored only in young children, using a number of imitation tasks that evaluated this spontaneous tendency in young children developed by Hannula and Lehtinen (2005). In these tasks, some numerical information is presented by the experimenter, but at no point is this information explicitly marked as important, and special care is taken to avoid any wording which could suggest that the tasks are mathematical or quantitative in nature. In these tasks, children have been found to differ in their tendencies to spontaneously direct attention to numerosity: Some children would imitate the exact actions of the experimenter (e.g., feeding a puppet “sweets”) with no regard to numerosity, whereas others would imitate the action while also attending to the numerical information (e.g., feeding the puppet *the exact number* of sweets as given by the experimenter, or regarding it in other ways, such as counting the number of distributed sweets, asking if they could give more or fewer sweets than given by the experimenter, etc.). These individual differences, regarding numerical information, were found to be stable over age in childhood, as demonstrated by positive correlations for three testing times, at 4, 5, and 6 years of age (Hannula & Lehtinen, 2005).

A number of studies have demonstrated correlations between SFON and children’s mathematical abilities (Batchelor, Inglis, & Gilmore, 2015; Bojorque, Torbeyns, Hannula-Sormunen, Van Nijlen, & Verschaffel, 2017; Hannula & Lehtinen, 2005; Hannula, Lepola, & Lehtinen, 2010; Hannula, Räsänen, & Lehtinen, 2007; McMullen, Hannula-Sormunen, & Lehtinen, 2015). For example, children’s mathematical abilities at the age of 3.5 years predicted their SFON tendency at the age of 4 years, which in turn, predicted later mathematical abilities at the ages of 5 and 6 years, which could not be accounted for by insufficient enumeration skills, linguistic abilities, or difficulties in comprehending task instructions (Hannula & Lehtinen, 2005). Other studies showed that the spontaneous tendency to regard numerosity at kindergarten age predicted mathematical abilities over and above other cognitive skills, either two or six years later, when the children were in their second or fifth grade at school, but again did not predict their reading skills at this age (Hannula et al., 2010; Hannula-Sormunen, Lehtinen, & Räsänen, 2015; McMullen et al., 2015).

The variance that children demonstrate in their responses to imitation tasks raises the question of whether this predictive relationship between SFON and later mathematical abilities is due to SFON reflecting individual differences in sensitivity to numerical information and greater resolution of quantitative processing. We hypothesized that the tendency to spontaneously focus on numerosity will be correlated with better numerosity discrimination. Moreover, we hypothesized that such individual tendency to attend the numerosity dimension of the environment should be present also in adulthood. Therefore, in the present study, we tested this hypothesis both in children (Experiment 1) and adults (Experiment 2).

# Experiment 1: Children

In this experiment, we measured both the number of times each child used numerical information during two of the imitation tasks developed by Hanuella and Lehtinen (2005), as well as the child’s Weber ratio on the Panamath task. We hypothesized that SFON scores would be negatively related to Weber ratios, such that children with a higher SFON counts would show better discrimination between quantities (i.e., a smaller Weber ratios).

## Method

### Participants.

Participants were 51 preschoolers (19 boys) ages ranging from 3 years 1 month to 4 years 11 months ( 3 years 10 months, 5 months). All children had no history of developmental disability, learning disabilities or attention deficits. Participants were recruited via WIZO (“Vitzo”) day care centers in Beer-Sheva through an invitation letter distributed by the kindergarten teachers, as well as by word of mouth. Parents who expressed interest in the study were contacted by phone and invited to participate in the study in two sessions.

Of the 51 children, data from 2 children was discarded due to them considered outliers on the Panamath task (see details below). Thus, data from a total of 49 children (17 boys) ages ranging from 3 years 1 month to 4 years 11 months ( 3 years 9 months, 5 months) was used in the final data analysis.

### SFON imitation tasks.

Two sets of imitation tasks were used to assess individual SFON scores. Both tasks were carried out in a quiet room either in our lab or in the child’s day care center. Throughout the imitation tasks, the experimenter avoided using any wording which might suggest that the tasks were mathematical or quantitative. The tasks included only small numerosities within the subitizing range (1-4), which all children should be able to handle.

In the first task, a toy parrot capable of “swallowing” was placed on the table in front of the child, and a bowl of small colorful stone “candies” (~3 cm in diameter) were placed in front of the parrot. The experimenter then introduced the child to Polly the parrot and said: “Watch carefully how I feed Polly, and then you do just like I did.” The experimenter then put two candies, one at a time, into the parrot’s mouth, where they disappeared into the parrot’s stomach. The child was then told: ‘‘Now you do exactly like I did’’. After the child completed “feeding” the parrot, this procedure was repeated three more times with the following amounts of candies: one candy in the second item, two candies in the third item, and one candy on the final forth item.

In the second task, a toy dump truck with an open-box bed, a small plastic shovel, and a container full of gravel were placed on the table. The experimenter then said: “Watch carefully how I put gravel in the truck, and then you do just like I did.” The experimenter then scooped two scoops of gravel, one at a time, into the dump truck using the toy shovel. The child was then told: ‘‘Now you do exactly like I did’’. After the child completed filling the dump truck, this procedure was repeated three more times with the following amounts of scoops: one scoop in the second item, two scoops in the third item, and one scoop on the final forth item.

Tasks were recorded using a digital camera placed out of the child’s sight, and recordings were used to code SFON scores. The child was scored as focusing on numbers on each item, if she or he imitated the correct numerosity, and/or if she or he was observed presenting any of the following quantifying acts: (a) utterances including number words (e.g., “I’ll give him three candies”), (b) use of fingers to express numbers, (c) counting acts, like whispering number word sequences and indicating acts by fingers, (d) other comments referring either to quantities or counting (e.g., “I miscounted”), or (e) interpretation of the goal of the task as quantitative (e.g., “I gave exactly the right number of candies”). The maximum combined score on both SFON tasks was 8.

When measuring SFON scores’ equivalent forms reliability between the imitation tasks (Webb, Shavelson, & Haertel, 2006) – comparing the scores calculated based on the first imitation task (Parrot) with those based on the second imitation task (Truck) – a reliability of 0.698 was found, indicating that children were consistent in their tendency to imitative the numerosity aspect between both tasks.

### Panamath.

Each participant’s threshold of discrimination (i.e., individual Weber ratios) was assessed using the Panamath task (Halberda et al., 2012). This task is a computerized dot-discrimination task (review and experience the task at <http://panamath.org/>). In each trial, two arrays of dots, one of green dots and one of orange dots, are presented side by side, and participants must judge which of two dot arrays is more numerous (see Figure 1). The difficulty of the task was adaptive and based on the participants’ performance, with the ratios getting smaller (making discrimination harder) after correct trials, and larger (making discrimination easier) after incorrect trials. After each selection, a feedback sound was played indicating if the child selected correctly or incorrectly. The next trial only began after the experimenter made sure the child was attending the screen.

Each participant preformed 4 minutes’ worth of trials (one block of roughly 130 trials) and a personal Weber ratio[[1]](#footnote-1) was calculated based on performance throughout the task. Individual Weber ratio scores reflect the ratio of change in numerosity each individual requires to be able to discriminate between the numerosities on 75% of the trials (with chance at 50%). For example, when presented with one array of 8 dots, an individual with a Weber ratio of 1.6 would need the second array to have a numerosity greater than to be able to discriminate between the arrays 75% of the time. Since Weber ratios are on a logarithmic-scale, the natural log of the ratios was used in all analyses.

Data from 2 participants with extreme Weber ratios (20.5, 5.33), indicating inattentiveness to the task, was excluded from analysis.

### Procedure.

For children recruited though WIZO (“Vitzo”) day care centers, both sessions took place at the day care center. For children recruited through word of mouth, the first session took place in our lab, and the second session took place at the child’s home. On the first session, after parents signed an informed consent from, the experimenter sat with the child in a quiet room, there the completed both SFON imitation tasks in sequence. Parents who came to our lab were compensated with 100 NIS. On the second session, with took place within two weeks of the first session, the experimenter sat with the child in a quiet room, there the child completed the computerized Panamath task on a portable PC.

## Results

All statistical analyses were conducted using R (version 3.4.0; R Core Team, 2016) in RStudio (version 1.0.143; RStudio Team, 2016).[[2]](#footnote-2)

The relationship between SFON and Weber ratios was examined using repeated-measures logistic regression, with SFON counts as the dependent variable, and the natural log of Weber ratios as a predictor. This method was chosen since logistic regression models are more suitable than linear regression models when analyzing count variables, and also have the added benefit of having greater statistical power (Jaeger, 2008; Warton & Hui, 2011).

A decrease in Weber ratios was found to predict an increase in the probability to spontaneously attend to numerical information in the SFON task, such that an individual with a discrimination ratio of 1:2 had a probability of 38% to attend to numerical information, while an individual with a discrimination ratio of 1:4 had a probability of 43% (, , ; see figure 2). Because age was related to SFON scores (, ), we examined the relationship between SFON and Weber ratios which controlling for age by including age as a covariate in the logistic regression, and found the relationship to remain significant (, ).

## Conclusion

As hypothesized, a negative relationship was found between SFON tendencies and numerosity discrimination among kindergarten aged children, indicating the children who are better at discrimination between numerosities have a higher tendency to spontaneously focus on numerical information, even when controlling for age.

The directionality of the relationship between SFON tendencies and numerical discrimination is not yet clear. It is possible that higher numerical processing abilities, as indicated by better numerosity discrimination, make numerical information more salient and thus children are more likely to integrate numerical information into their behaviors. However, it also possible that SFON tendencies improve numerical processing in some manner.

# Experiment 2: Adults

As mentioned, the extant research on SFON has focused on this tendency in children. The SFON imitation tasks have been developed for children and are not suitable for adults. We therefore designed a computerized numerosity bias task (NBT) that would allow us to measure adults’ SFON, while adhering the same criteria laid out by Hannula and Lehtinen’s (2005) children’s SFON tasks: (1) The task would have no explicit request to regard numerosity information as relevant, but would (2) include numerosity information that could be processed if attended to, along with other information that would compete with the numerical aspect of the task.

Using this task, we investigated the adults’ tendency to spontaneously regard numerical information. We hypothesized that, similar to findings among children, adults would demonstrate variance in their SFON tendency. Moreover, we hypothesized that these individual differences would be associated with mathematical abilities; specifically, with discrimination ratios, as well as with mathematical achievements, as measured by mathematical psychometric scores and math matriculation levels. Finally, we hypothesized that as with children, SFON tendency is a specific mathematical marker, and thus would not be correlated with general intelligence. Such findings would indicate that SFON is a perceptual bias that not only plays a role in childhood, but is also apparent in adulthood. Moreover, if adult SFON tendencies were found to correlate with other mathematical abilities in adulthood, it would strengthen SFON’s status as a specific-marker for mathematical abilities.

## Method

### Participants.

Participants were 84 students ( years, ; 29 males) at Ben-Gurion University of the Negev. All subjects had normal or corrected-to-normal vision. Participant received 50 NIS or course credit in exchange for their participation in the study.

Data from 7 participants was discarded due to them considered outliers on the Panamath task (see details below). Thus, data from a total of 77 participants ( years, ; 26 males) was used in the final data analysis.

### Numerical bias task.

In this computerized force-choice task, participants were instructed to learn for each block which of two stimuli was the one that earned them points as indicated by feedback following their selection. In each block, the selection of one stimulus would award them points, while the selection of the other stimulus would result in a deduction of points. Participants were instructed to select one of the two stimuli by pressing either a button on the right with their right index finger to select the right stimulus, or a button on the left with their left index finger to select the left stimulus. Responses were collected using a serial response box (SRBox). Participants were instructed to respond as quickly as possible.

Each block consisted of 8, 9, 10 or 11 trials. Each trial began with a fixation cross appearing for 500 ms, followed by two stimuli presented on a black background separated by a gray vertical line. The two stimuli always differed from one another on two dimensions: the number of objects comprising the stimulus (1, 2, 3, or 4) and objects’ color (red, green, blue, or yellow). For example, the two stimuli could be three red dots and two yellow dots (see Figure 3). The number of objects and colors used to comprise the stimuli were randomly selected and differed between blocks.

Unbeknownst to the participants, each block was divided into two phases: a learning phase that consisted of all but the last trial, and a test phase that consisted of the last trial. In the learning phase trials, the stimuli were presented until a response was made or for a maximal duration of 1,500 ms, followed by feedback indicating if points were won or lost. Throughout the trials in the learning phase (i.e., all but the last trial of each block), color and numerosity were consistently paired, such that, for example, the three dots were always red, and the two dots were always yellow. This allowed participants to learn fairly quickly, through trial-and-error, which stimulus earned them points (e.g., 2-red; see Figure 3A). Because the two stimuli differed on two dimensions, the manner in which participants identified the rewarding stimulus could be based on either the color of the stimulus (“choosing red awards me points”) or on the number of items comprising the stimulus – its numerosity (“choosing three dots awards me points”). Both strategies would lead to the same performance because color and numerosity were paired throughout these trials in each block.

The last trial in each block was designed to test whether identification was based on color or numerosity. This was done by reverse-pairing the two dimensions; for example, if the stimuli in the leading trials were 3-red vs. 2-yellow, the stimuli in the final trial would then be 2-red vs. 3-yellow (see Figure 3B). If the identification was color-based, the participant would, in this final trial, select the stimulus comprised of the color that previously awarded points (e.g. 2-red), but if learning was number-based, the participant would select the stimulus comprised of the numerosity that previously awarded points (e.g. 3-yellow). This allowed us to measure which dimension (numerosity vs. color) was more salient to the participant and thus assess the participant’s SFON. In this last trial, the stimuli were presented until a response was made or for a maximal duration of 3,000 ms. No feedback was given for these test trials.

To control for the possibility that our measure could be contaminated by a bias against attending to “color”, we separately measured the participant’s spontaneous focusing on color. This was measured using 8 additional blocks in which the stimuli were also comprised of two dimensions: color and shape (e.g., blue-triangle vs. red-square), with the number of objects kept constant at 1. In total, each participant completed 32 blocks: 24 color-vs-numerosity blocks and 8 color-vs-shape blocks. A short break was given after every 8 blocks.

Throughout the entire task, the side (right or left) of the rewarding stimulus was balanced and randomly selected between trials for both the learning and test trials. Moreover, perceptual variables such as objects’ size and location on the display were controlled and balanced between trials; the average display area that was colored was equal between each of the possible number of objects and the different shapes (approximately 4,700 pixel2), and the location of each object was selected at random, inasmuch as the object’s location did not overlap another object, and the objects of each stimulus were confined to one side of the screen. The task was designed in E-Prime software (released candidate 2.0.8.9; Psychological Software Tools Inc., 2010).

Individual SFON scores were calculated as the proportion of test trials in which each participant chose according to the numerical dimension as opposed to the color dimension in the first 24 blocks (the color-versus-numerosity blocks). Similarly, individual color-bias scores were calculated as the proportion of test trials on which each participant chose according to the color dimension as opposed to the shape dimension in the last 8 blocks color-versus-shape blocks.

### Mathematical achievements.

Participants was asked to report their quantitative reasoning score (QRS) from their psychometric entrance test (PET, equivalent to SAT test) scores. These are standardized scores ranging between with an average score of points, . The QRS are derived from the quantitative reasoning section, which tests the ability to use numbers and mathematical knowledge in solving quantitative problems presented in verbal form but also in visual form such as tables and graphs (National Institute for Testing and Evaluation, n.d.).

Participants were also asked to report the level (ranges between 3-5) they achieved on the mathematical Bagrut (matriculation) exam, which is completed in high school.

Of the XX participants, XX (XX%) did not provide their QRS or math matriculation level (XX) or provided impossible data (outside the range of possible values).

### Panamath.

Discrimination ratios were measured using the adult version of the Panamath task described in experiment 1. Participants preformed 2 practice trials, in which feedback was given for selecting correctly or incorrectly. Participants then preformed 8 minutes’ worth of trials (one block of roughly 264 trials). Feedback was not given on these trials.

Data from 7 participants was excluded from analysis due to their Weber ratios falling above their age-appropriate 90th percentile (data for normal range is provided with the Pnamath softwere; Halberda et al., 2012).

### Raven.

General intelligence was tested using the Raven Standard Progressive Matrices (Raven, 1960). Participants were given 30 minutes to complete 36 matrices. Raven scores were calculated as the percent of correct answers throughout that task.

### Procedure.

When signing up for the study, the study’s description contained no reference to numerosity or mathematics, as to avoid participants arriving at the lab primed to numerical thinking in any way. After giving written consent, each participant was seated approximately 70 cm from a computer screen and was given the NBT task instructions, which contained no explicit request to regard numerosity information, only that participants must learn which stimuli is the “correct” one. Participants then completed 10 practice trials (that were not used in the final analysis). After making sure that the task instructions were understood, the experimenter left the room and participants completed the rest of the task.

After completing the NBT task, participants were asked to report on their past mathematical achievements (Quantitative Reasoning score and Bagrut level in mathematics). Participants then completed the Raven test and the Panamath task. When all tasks were completed, participants were thanked for their time and given 50 NIS of course credit.

The order of the tasks was deliberate, to ensure that the subjects did not perform any numerical task prior to the NBT that could serve as an unwanted priming cue.

## Results

Individual SFON scores, ranged from 0 (responses were never based on the quantitative dimension of the stimuli) to 1 (all responses were based on the quantitative dimension of the stimuli), with a median score of 0.13. When measuring the reliability of these scores using the Spearman–Brown method (Webb et al., 2006) – comparing the scores calculated based on the odd blocks with those based on the even blocks – a reliability of 0.969 was found, indicating that adult participants were consistent in their tendency to spontaneously regard numerical information throughout the task.

As expected, spontaneous focusing on color was correlated with SFON (, ), therefore it was statistically controlled for in all analysis of SFON by adding it as a predictor.

To test whether individual differences in SFON among adults were related to numerical discrimination, SFON counts were subjected to a logistic regression analysis with Weber ratios as a predictor. As hypothesized, a decrease in Weber ratios was found to predict an increase in the probability to spontaneously attend to numerical information in the SFON task, such that an individual with a discrimination ratio of 1:4 had a probability of 16% to attend to numerical information, while an individual with a discrimination ratio of 1:8 had a probability of 39% (, , ; see figure 4).

Two logistic regression analyses where conducted to test the relationship between SFON and QRS and between SFON and matriculation level. Since a gender difference was found in QRSs (, ) and a marginally significant gender difference was found in matriculation level (, ), gender was also included as a predictor in both analyses. As predicted, adults with higher QRSs scores also had higher odds of attending to the stimuli’s numerosities, such that an increase in QRS from 100 (the mean) to 120 (one standard deviation above the mean) increased the probability of attend to numerical information from 7% to 17% (, , ). Likewise, adults who completed a higher level of matriculation in math showed higher odds of attending to the stimuli’s numerosities, such that an of matriculation level from 3 to 4 increased the probability of attend to numerical information from 18% to 24% (, , ).

Finally, we tested whether SFON is related to Raven scores. As predicted, Raven scores were not found to have any predictive power (, , ).

## Conclusion

Similar to previous results among children, attentional bias toward numerical information in adults was associated with mathematical abilities as well as mathematical achievements. Individuals who scored higher in SFON were better able to discriminate between numerosities. Additionally, SFON scores were also associated with two measures of mathematical achievement. Together with the lack of association between SFON scores and general intelligence, these results seem to indicate that – similar to the results found with children – SFON in adults is a domain-specific marker.

# General Discussion

The present study investigated the relationship between the tendency to spontaneously focus on numerosity (SFON) and numerical discrimination, both in children and adults. In preschool children, SFON was measured using Hannula and Lehtinen’s (2005) imitation tasks. In adults, SFON was measured using a novel computerized Numerical Bias task. We found that in both age groups that numerosity discrimination was related to SFON, such that individuals with greater resolution of quantitative processing were more likely to spontaneously attend to quantitative information. Furthermore, in adults, SFON was also found to be related to mathematical achievements.

Our findings support the notion that the tendency to spontaneously focus on numerosity is due to greater resolution of quantitative processing which increases the saliency of quantitative information. Previous work has demonstrated that the degree to which numerical information is prominent affects the likelihood of incorporating this information into behavior. For example, Cantlon, Safford, and Brannon (2010) found that children spontaneously matched between visual stimuli based on numerosity, as opposed to surface area, and more importantly, that their bias towards numerosity-based matching was affected by the numerical ratio between the two visual stimuli. Similarly, when examining the distribution of responses in a SFON imitation task, the size of the errors made were proportionate to the trial-relevant numerosity (Sella, Berteletti, Lucangeli, & Zorzi, 2016); i.e., as the trial-number grew, children were less precise in their imitation of the numerosity used by the experimenter. Over all, greater resolution in quantitative discrimination seems to affect the saliency of quantitative information, which in turn increases the probability of incorporating said information into decision making and ultimately behavior.

Another hypothetical explanation to this association could be an influence of SFON tendency on the acuity of numerical discrimination. Previous studies of SFON had found evidence of reciprocal relation between SFON and early numerical skills, indicating that children’s self-initiated practice in focusing on exact numerocity and the incorporation of it into everyday situations facilitates enhanced mathematical performance, and vice versa. Essentially, the idea is that SFON tendency causes the triggering of exact number recognition, facilitating numerical fluency (Hannula et al., 2010), possibly including the acuity of their analog nonsymbolic number representation. This suggested relation is in line with studies showing quantitative discrimination could be enhanced by training (Park & Brannon, 2013; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006).

To our knowledge, we provide here the first evidence for the existence of individual differences in SFON among adults, using the novel NBT task we developed for measuring SFON in adults . This finding suggests the possibility that SFON could be a stable bias, continuing from childhood and into adulthood. This notion is consistent with previous studies which found stability in SFON throughout childhood (Hannula & Lehtinen, 2005). Similar findings regarding attentional biases and its stability come from the field of clinical psychology, where such attentional biases have been found to be stable from a young age (Gupta & Kar, 2012). For example, individuals high in anxiety are abnormally sensitive to threat-related stimuli and tend to direct their attention toward threatening information during early automatic stages of processing (Williams, Watts, MacLeod, & Mathews, 1988). Still, the stability of SFON as an individual characteristic from childhood to adulthood was not directly assessed in this study and would have to be explored in future research.

Our study offers further innovative findings to the growing research of SFON tendency and its role in the development of numerical abilities. There are several core open questions remained in it, namely what is the mechanism underlying individual differences in SFON tendency, and how does this tendency lead to later mathematical advantages. Although a growing body of evidence, across different cultures and ages confirm further the domain specific role of SFON tendency in mathematical achievements throughout development, no particular set of factors (i.e, executive function, IQ, home and school environment) that were tested could sufficiently explain the variability in SFON tendency found in children, nor fully account for its relation to later on mathematical achievements (see review in Rathé, Torbeyns, Hannula-Sormunen, De Smedt, & Verschaffel, 2016). Here we provide a novel insight regarding the mechanism of SFON, as we show the first evidence that higher tendency of SFON is associated with the acuity of the AMS, an innate, domain specific cognitive foundation of the ability to nonverbally represent and manipulate quantities, and also high-level mathematical abilities (Park & Brannon, 2013; Wilson et al., 2006). The AMS and the SFON tendency have much in common as factors in numerical processing development; individual differences in both factors are evident from early childhood, and these variations are predictive of later mathematical competence (or lack of it, relating to core mathematical deficits, e.g., Kucian et al., 2012; Wilson et al., 2006) in both cases. Notably, they both manifest independently of external intervention, although both can be influenced by such. In light of all these findings, we suggest that the individual sensitivity to quantities (derived from individual AMS acuity) constitutes a prominent candidate for the mechanism underlaying the development of SFON tendency and its implications on subsequent mathematical abilities.

Yet these conclusions must be taken with careful consideration to some limitations of our study. Our findings are not only consistent with previous findings in this field, but also show conceptual replication of the relation found between AMS acuity and SFON in children and adults by two different techniques measuring SFON tendency, offering converging evidence. Still, the extent to which the SFON measured in the NBT is theoretically identical to the SFON measured in the imitation tasks cannot be determined based on our study. This can be examined in a longitudinal study in which children’s SFON scores measured from the imitation tasks are able to predict adult’s SFON scores from the NBT. Additional dissimilarities between the adult and the children SFON task may add to this uncertainty, such as number of trails (8 trails for children, ##), one task for adults while children were measured in two imitation tasks, and different scales in dependent variable measure (). That being said, the stated variations between the tasks are largely due to differences in numerical fluency, working memory load and adequate task complexity between the age groups (תכלס לא בטוחה בכלל אם זה נכון, לא מצאתי בפירוט המטלה מה היה טווח הכמויות שהשתמשת בהן), and provided opportunity to validate (to some extent) the NBT task. Furthermore, in both tasks we measured an implicit process, clear of interference by utilization of different strategies or task directions. Finally, the directionality of the relationship between SFON and the AMS acuity cannot be determined in this study, and all discussion regarding such in this paper is purely speculative.

There is an ongoing debate in the extant literature, regarding the role of nonsymbolic numerosity perception in the development of symbolic numerical and mathematical skills (REF). For example, XX found no relationship… were as XX found that…. Our results suggest that spontaneously attending nonsymbolic quantities information does have implications for mathematical skills, as seen in SFON’s relation to mathematical achievements found here among adults.

יש דיון בספרות לגיב המשמעות של תפישה לא סימבולית של כמויות להצלחה בעולם המספרים ומתמטיקה – לא ממש מוצאים את הקשר הזה. יש בספרות שאלה לגבי מידת המשמעות של הבדלים בעיבוד כמויות לא סימבוליות להצלחה במתמטיקות. המחקר הנוכחי מרמז על כך שההפניה הספונטנית לעולם הכמותי יש אימפליקציה ליכולות מתמטיות. ANSARI לא מצא.

חוזקה: זה חדשני, טרם נבדק – גם מבחינת הקשר וגם מבחינת המטלה שפותחה לצורך העניין במבוגרים. יש פה רפליקציה קונספטואלית בשתי מתודולוגיות שונות גם בילדים וגם במבוגרים – SFON נמדד בשתי שיטות שונות (converging evidence). יש פה מדידה של תהליכים אימפלציטיים – לא תלוי בהוראות, או באסטרטגיה מיוחדת. יחסית מדידה עדינה ונקיה.

Still, our study suffers from several methodological limitations. First, it is unclear to which extent the SFON measured in the NBT is the theoretically identical to the SFON measured in the imitation tasks. This can be examined in a longitudinal study in which children’s SFON scores measured from the imitation tasks are able to predict adult’s SFON scores from the NBT. Additionally, when measuring SFON using the imitation tasks, the final score is based on relatively few trials (8 trials). Yet, מהימנות גבוהה

חולשות: האם מדדנו את אותו מבנה תאורטי של SFON בין ילדים למבוגרים. המדידה מבוססת על יחסית מספר מועט של צעדים. במבוגרים זו רק מטלה אחת, ואילו אצל הילדים זה שתיים. המדגמים הם לא מאד גדולים בילדים. המדידה של הSFON היא מדידה איכותנית ולא כמותית – המשתנה הוא לא בדיוק רציף.

The directionality of the relationship between SFON and discrimination is undetermined.

In conclusion, ….

Acknowledgements

We would like to acknowledge the contributions of Ms. Anat Feldman, Ms. Maya Bergman and Ms. Morielle Holtzman throughout the study, without whom this research would not have been completed.

Compliance with Ethical Standards

The authors declare that they have no conflict of interest. This work was partially supported by the Israeli Science Foundation grant no. 1799/12 awarded to the Center for the Study of the Neurocognitive Basis of Numerical Cognition. All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards. This article does not contain any studies with animals performed by any of the authors. Informed consent was obtained for all individual participants included in the study.

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Figure Captions



*Figure 1.* SamplePanamath trial for children. In this trial, the green array contains 12 dots and the orange array contains 5 dots. Individuals with discrimination ratios greater than 2.4 would be able to detect that the green array is more numerus.

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*Figure 2*. Children’s SFON scores as a function of their Weber ratios. The black line is the estimated logistic-regression line.



*Figure 3*. Example number versus color block. (A) Example learning trial, with the right stimulus comprised of 3-red dots and the left stimulus comprised of 2-yellow dots. Feedback was given according to selection, allowing participants to learn to select 3-red dots. (B) The last trial in each block was the test trial. In this final trial, color and numerosity were reverse-paired, producing the stimuli 2-red dots and 3-yellow dots. No feedback was given for this trial.

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*Figure 4*. Adults’ SFON scores as a function of their Weber ratios. The black line is the estimated logistic-regression line.

1. The Panamath task returns Weber fractions, which were transformed to Weber ratios via (Halberda, Mazzocco, & Feigenson, 2008). [↑](#footnote-ref-1)
2. All data and R code used for analyzing the data are available online at <https://osf.io/gmuv5/?view_only=82e37668c61f4b04ad5c5d79682373a9> [↑](#footnote-ref-2)